

Crystallographic Data and Model Quality 2

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Abstract 4

This article gives a consistent classification of sources of random and systematic errors in crystallographic data, and their influence on the averaged dataset obtained from a diffraction experiment. It discusses the relation between precision and accuracy and the crystallographic indicators used to estimate them, as well as topics like completeness and high-resolution cutoff. These concepts are applied in the context of presenting good practices for data processing with a widely used package, XDS. Recommendations are given for how to minimize the impact of several typical problems, like ice rings and shaded areas. Then, procedures for optimizing the processing parameters are explained. Finally, a simple graphical expression of some basic relations between data error and model error is suggested.

Key words X-ray crystallography, Accuracy, Precision, Random errors, Systematic errors, Merged data, Unmerged data, Indicators 13 14

1 Introduction 15

In the last decades, crystallography has been highly successful in delivering structural information about proteins, DNA, and RNA, the substrates of life on earth. The resolution of the method is good enough to discern the three dimensional structure of these macromolecules at the atomic level, which is essential to understand their diverse properties, functions and interactions. However, although it is easy to calculate the diffraction pattern for a given structure, the reverse task of deriving a molecular structure from just a single set of unique diffracted intensities is difficult, as the mathematical operation governing the former direction cannot be inverted in a unique way. To be solved experimentally, this “inverse problem,” or more specifically “phase problem,” requires more than just a single set of unique diffraction data. High quality of the data is a requirement for the experimental solution of the problem, but also for the refinement of the macromolecular structure, as discussed in Subheading 4. 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

32 The correct biological interpretation requires the best possible
33 model of the macromolecule. To obtain the best model, every step
34 of the structure determination procedure has to be performed in a
35 close-to-optimal way. This means that the purification of the mac-
36 romolecule, its crystallization, crystal handling, measurement of
37 diffraction data, processing of the resulting datasets, and down-
38 stream steps such as structure solution, refinement, and validation
39 each constitute scientific tasks that deserve specific attention, and
40 have been undergoing continuous enhancements throughout the
41 history of macromolecular crystallography.

42 Two kinds of numerical data are the result of a crystallographic
43 experiment and usually deposited as such in the Protein Data Bank:
44 the diffraction intensities as a reduced representation of the diffrac-
45 tion experiment, and the atomic coordinates resulting from the
46 visual inspection and interpretation of electron density maps, and
47 subsequent refinement. A third kind of numerical data, the raw
48 data (frames) obtained in the diffraction experiment, have so far
49 not been usually deposited in long-term archives, mainly due to
50 (disk) space concerns. This is unfortunate since archiving of raw
51 data would enable reprocessing of incorrectly processed data as
52 well as enabling and taking advantage of future improvements in
53 methodology, like extracting the diffuse scattering information.

54 The discussion focuses on data that correspond to a *single*
55 atomic model. This rules out all the complications that arise from
56 merging of non-isomorphous datasets, where each individual data-
57 set corresponds to a different model—in this situation, a merged
58 dataset would represent something like an average model, which
59 violates the physicochemical requirements, and may not be bio-
60 logically meaningful.

61 This chapter first presents the principles and concepts that
62 need to be understood in the context of the rather broadly used
63 term “data quality”; similar presentations may be found for exam-
64 ple in refs. [1–3]. Second, the application of these principles to
65 data processing with the XDS program package [4, 5], which the
66 author is most familiar with, is explained. Third, data and atomic
67 model are related in a graphical way, which allows some important
68 and nontrivial conclusions to be drawn about how the former
69 influence the latter.

70 2 Errors and Crystallographic Indicators

71 The goal of a crystallographic experiment is to obtain accurate
72 intensities $I(hkl)$ for as many Bragg reflections hkl as possible. I
73 discuss two kinds of errors, random and systematic error, which
74 exist in any experiment. A major difference between them is that
75 the relative error arising from the random component *decreases*
76 with increasing intensity, whereas the relative error in intensity

from the systematic component is (at least on average) *constant*, 77
often in the range between 1 and 10 %. A more specific descrip- 78
tion for systematic error would thus be “fractional error,” but this 79
name is not in common use. Nonlinear errors also exist, but play 80
a minor role. 81

A well-designed crystallographic experiment has to strike an 82
appropriate compromise between the two kinds of error. For 83
example, a reduction of random error (see below) can be obtained 84
by longer or stronger exposure of the crystal, but this will inevita- 85
bly increase the systematic error from radiation damage to the 86
crystal. Ideally, the sum of both errors should be minimal, and 87
programs, e.g., “BEST” [6], exist that suggest a compromise, in 88
the form of a proposed “strategy” for the experiment. Fortunately, 89
the gradient of the sum of both errors is close to zero at and near 90
the optimal strategy, which means that small deviations from the 91
optimal strategy do not substantially decrease data quality. 92

The discussion of errors has to take the distinction between 93
precision and accuracy into account. The term “precision” refers 94
to the reproducibility of an experiment, and to the internal consis- 95
tency or relative deviation of the values obtained. For example, if 96
the number $\epsilon = 2.718\dots$ should be determined in an experiment, 97
and two measurements would yield the values 3.217 and 3.219, 98
then these measurements are considered precise, because they 99
agree well with each other—their relative deviation is small. 100
However, they are not close to the true value—the error (or inac- 101
curacy) in their measurement amounts to about 0.5. 102

The term “accuracy,” on the other hand, refers to the devia- 103
tion of measured values from the true values. In this example, if 104
two measurements would yield the values 2.6 and 2.8, then the 105
results from this experiment are more accurate than that from 106
the previously mentioned experiment, although they are not as 107
precise. 108

Optimizing an experiment for precision alone therefore does 109
not ensure accuracy; rather, equating accuracy with precision also 110
requires the absence of any kind of error that has not been taken 111
into account in the precision estimate. To estimate accuracy, we 112
thus need to quantify both the precision of the data, and the unde- 113
tected error (which usually requires some knowledge about the 114
true value obtained by other means). If both can be quantified, we 115
can estimate the accuracy as the absolute or relative error of a 116
measurement. 117

2.1 Random Error

The crystallographic experiment measures the number of photons 118
contributing to each detector pixel. These photons arise from 119
Bragg reflections, but also from background scatter. The number 120
of photons in each pixel is subject to random fluctuations. These 121
are due to the quantum nature of photons; there exists a certain 122
probability of emission of a given photon by the crystal into a given 123

124 pixel in a unit of time, and each photon's emission into that pixel
125 is independent from that of other photons. As a result, photon
126 counts are governed by Poisson (counting) statistics, which math-
127 ematically means that the variance of the photon number is equal
128 to the photon number itself. Furthermore, a CCD detector may
129 contribute a random component ("read-out noise") to the total
130 photon count (pixel detectors are almost noise-free), which is also
131 due to quantum fluctuations in the detector hardware and may be
132 considered as additional background.

133 Data processing software essentially adds the counts of the pix-
134 els belonging to each reflection, and subtracts an estimate of the
135 background in each pixel, to give $I(hkl)$, the intensity of the Bragg
136 reflection. The variance of $I(hkl)$ may be calculated, using the rules
137 of error propagation, from the known variances of each contribu-
138 tion to $I(hkl)$; its square root will be called $\sigma_0(hkl)$ in the following.
139 For strong reflections, where the background is negligible, this
140 procedure gives a precision, expressed as relative random error in
141 $I(hkl)$, of $\sigma_0(hkl)/I(hkl) \sim 1/\sqrt{I(hkl)}$.

142 The relative amount of the random error may be reduced by
143 repeating the experiment, and averaging the results of the indi-
144 vidual experiments. As the laws of error propagation show, the pre-
145 cision of the estimate of the averaged intensity is improved by a
146 factor of \sqrt{n} over that of an individual measurement, if n is the
147 number of repeated experiments with independent errors. Thus,
148 the precision of the averaged (also called "merged") data may be
149 high even if the precision of each individual observation is low.

150 The square root function appears both in the relative error of
151 a photon count arising from Poisson statistics, and in the improve-
152 ment of precision from averaging of multiple measurements. It is
153 important to realize that the mathematical reasons for the occur-
154 rence of the square root differ. Nevertheless, the fact that the
155 square root occurs in both situations means that the relative error
156 is in principle the same whether a reflection is measured ten times
157 and averaged, or measured just once, but with ten times stronger
158 exposure or ten times as long.

159 A high number (multiplicity, sometimes called redundancy) of
160 observations of each unique reflection, together with low exposure
161 of each observation, is therefore equivalent in terms of the preci-
162 sion of the merged data, to an experiment in which each unique
163 reflection is just measured once, but exposed proportionally stron-
164 ger. Thus, if only random error is considered, there would be no
165 reason to perform experiments with high multiplicity.

166 **2.2 Systematic Error**

167 The term "systematic error" summarizes all types of error that are
168 not purely random in nature, and these are due to macroscopic
169 physical or technical properties of the experimental setup, the crys-
170 tal, and the processing of its data. For instance, systematic errors
may arise from imperfect spot shapes (split crystal), radiation

damage, absorption differences due to crystal shape and mounting, shutter synchronization problems, imperfect detector calibration and inhomogeneity of detector sensitivity, shadowed parts of the detector, nonlinear or overloaded detector, vibrations for example due to the cryo stream or fluctuations of the primary X-ray beam, imperfect or inaccurate assumptions about geometric parameters and computational models applied in the data processing step, and to other problems that may be significant for a given experiment.

Systematic error may appear to be random if its cause is unknown or cannot be fully described or modelled, but contrary to random counting error, the change of a reflection's intensity is usually (at least on average) proportional to the intensity itself—thus the term “fractional error.” For example, a fluctuation in beam intensity changes all intensity values by the same percentage; absorption in the crystal or loop changes intensities in proportion to their original value; and using, during data processing, a mosaicity value that is too low, or a summation (integration) area that is too small, will chop off a certain fraction of the intensity.

Contrary to random error, the relative error of a single observation is *not* decreased by higher flux or longer exposure. However, many kinds of systematic errors in a crystallographic experiment at least partially cancel out if multiple measurements are averaged. This is the case if the experiment samples the possible values of the error term multiple times, in an even (or at least random) and unbiased way. Examples are beam instability, shutter problems, and most aspects of detector non-ideality, except those that result in nonlinear response (e.g., overload). Their influence on the final averaged data is decreased by averaging of n independent observations, and indeed the reduction of error then follows the same \sqrt{n} rule as applies to random error.

These kinds of systematic error may thus be considered as benign: their influence on the merged data may be mitigated by distributing the total experimental time and dose over many observations, or by collecting multiple datasets [7]. It is therefore systematic error, not random error, that mandates the collection of data over more than the absolute minimum of rotation range required for obtaining complete data. However, it is important to realize that after a full turn of the spindle, all those systematic errors that depend on the geometry of the experiment will be exactly repeated. It is thus highly advisable to change the crystal setting on the goniostat after at most 360° .

If all or most observations of a unique reflection are systematically affected in the same or a similar way, their systematic errors are not independent, and averaging may not necessarily decrease the systematic difference between true and estimated intensity (the accuracy). Known or well understood effects may often be modelled by analytical or empirical formulas. If a model for the specific error type is available and appropriate, the systematic difference is

219 accounted for, and any remaining difference between intensities
220 may become a useful signal. In this way, a systematic effect may
221 become a part of an extended description of the experiment, and
222 does no longer contribute to the experimental error.

223 An example for this is absorption by the crystal and its environ-
224 ment (loop, mother liquor)—if it can be properly modelled, its
225 influence is compensated. However, in low-symmetry space
226 groups, all symmetry-related reflections may systematically be
227 weakened or strengthened in the same way. Since only those sys-
228 tematic errors that lead to systematic differences can be corrected,
229 no information about the proper absorption correction is available
230 in this case. Therefore, at least one additional dataset should be
231 measured in a different orientation of the crystal. The systematic
232 absorption difference between the two resulting datasets may then
233 be detected and corrected in software. It should be noted that even
234 if absorption is not corrected in the data processing stage, it can be
235 approximately compensated by an overall anisotropic overall dis-
236 placement parameter in the refinement stage. This parameter then
237 should not be interpreted as its name suggests, but rather as a com-
238 pensation factor for an experimental property.

239 Importantly, for strong reflections (low resolution), systematic
240 error is usually higher than random error; the converse is true for
241 weak reflections (high resolution), where the signal-to-noise ratio
242 is usually dominated by the random error term. However, radia-
243 tion damage, the most devastating kind of systematic error, is an
244 exception to this rule. Radiation damage, which changes (and ulti-
245 mately destroys) the structure of the macromolecule during the
246 measurement, induces a systematic error that is not mitigated by
247 averaging of multiple observations, because it results in intensity
248 measurements that do not scatter around a true value, but rather,
249 with increasing dose, deviate further and further from the true
250 value—the intensity at the beginning of the experiment.

251 The detrimental influence of radiation damage has to be
252 avoided to a degree that depends on the kind of experiment, and
253 its desired goal. In recent years, there has been some progress in
254 describing the relation between dose and its footprint on the mac-
255 romolecule [8, 9]. Furthermore, the influence of radiation damage
256 may be partially compensated by zero-dose extrapolation, a com-
257 putational technique [10]. However, it should be noted that the
258 relative change of intensities by radiation damage is biggest at high
259 resolution, where the signal may be so weak (i.e., the individual
260 measurements so imprecise) that zero-dose extrapolation becomes
261 inaccurate.

262 **2.3 An Indicator** 263 **for the Systematic** 264 **Error**

265 The error estimates $\sigma_0(hkl)$ obtained during integration of observa-
266 tions, being only based on counting statistics, are lower than the
267 actual differences between intensities of symmetry-related observa-
268 tions, because the latter include the differences due to systematic

errors. To account for the full difference, and thus to arrive at more 266
useful error estimates $\sigma(bkl)$, the scaling and merging procedures 267
operating on multiple reflections inflate the estimated error using 268
an empirical formula [3, 11] that employs separate scale factors for 269
the random and the systematic error. This modification of error 270
estimates works quite well in practice, and the coefficients of the 271
formula (the “error model”) can be used to obtain an estimate of 272
the systematic error of the dataset. The resulting estimate can be 273
expressed as an “asymptotic signal-to-noise ratio,” abbreviated ISa 274
[11]. ISa gives the numerical value of I/σ for a hypothetical, infi- 275
nitely strong reflection in the dataset, after adjustment of its σ by 276
the new error model. If no systematic errors would exist, ISa would 277
be infinite, since σ would be, from Poisson counting statistics, just 278
the square root of I . However, real experiments are never ideal, 279
which is why ISa is finite. ISa does not depend on the random 280
error, making it insensitive to for example crystal size, mosaicity, 281
exposure time, and flux, and is thus on an absolute scale. 282

The author has processed many datasets from different syn- 283
chrotron beamlines and detectors. Empirically, it is found that 284
datasets from CCD detectors rarely have ISa bigger than 30; data- 285
sets from pixel array detectors may have ISa values about twice as 286
high compared to datasets from the same crystals collected on 287
CCD detectors. This means that the use of a pixel array detector at 288
a very stable beamline in an experiment with a good crystal may 289
result in down to half the systematic error, compared to a CCD 290
detector, and demonstrates the importance of detector technology. 291
Conversely, if a split crystal, or a dataset with strong radiation dam- 292
age, is measured on a perfect beamline, ISa may be as low as 10 293
(i.e., meaning that even the strongest reflections in the dataset will 294
have signal-to-noise less than 10). Likewise, a good crystal may 295
give a low ISa when the experimental setup or beam suffers from 296
instability, or the cryo stream makes the crystal vibrate. 297

Data processing programs implement simplified or idealized 298
assumptions about the experiment, and thus may themselves con- 299
tribute some systematic error. To investigate the magnitude and 300
properties of the systematic error from data processing, the author 301
wrote the program SIM_MX [12] that allows to simulate complete 302
datasets with specified amounts of random and systematic errors. 303
A simulated dataset with only random error should ideally produce 304
an ISa of infinity. However, the data processing program that the 305
author is most familiar with (XDS), in this situation gives ISa well 306
above 100. This means that if a good crystal is used to obtain a 307
dataset, the overall systematic error, as measured by ISa, is mostly 308
due to deficiencies of the experimental setup, and not due to short- 309
comings of the data processing program. Unfortunately, a general 310
weakness of the analysis of systematic error is that the simple error 311
model usually employed does not allow to identify and thus cor- 312
rectly model the exact source of systematic error. 313

314 Conceptually, it is almost always impossible to accurately
 315 measure the systematic error, since the true intensities are unknown
 316 and not measurable. Obviously, ISa *underestimates* the overall sys-
 317 tematic error, as only those systematic errors that lead to systematic
 318 differences enter its calculation. In my research group, we usually
 319 find that upon scaling two or more datasets together, the new error
 320 model for each dataset, which is calculated by the scaling program
 321 (XSCALE), almost always leads to a reduction of its ISa. This is
 322 due to the fact that only by comparing datasets can some system-
 323 atic errors be detected.

324 For isomorphous datasets (e.g., same crystal but different ori-
 325 entation) which are >90 % complete, have average multiplicity of 3
 326 and more, and have higher symmetry than triclinic, we find that
 327 the reduction of ISa is usually less than 10 %. Therefore, the under-
 328 estimation of the systematic error by ISa is usually minor.

329 If non-isomorphous crystals are scaled and merged and the
 330 error model is recalculated, the newly determined ISa values have
 331 to account for the systematic differences arising from differences in
 332 unit-cell parameters and crystal contents, and may then be much
 333 reduced.

334 **2.4 Indicators** 335 **for the Precision** 336 **of Unmerged Data**

337 In crystallography, indicators to describe aggregated properties of
 338 the data are necessary because the number of reflections is so large
 339 that it is prohibitive to inspect individual reflections. The availabil-
 340 ity of multiple observations (called “multiplicity” or “redundancy”)
 341 allows their precision to be measured.

342 Historically, R_{sym} was proposed by Arndt [13] for analysis of
 343 data from the first electronic area detectors. At the time, his inter-
 344 est was to create an indicator for how reproducibly the data are
 345 measured with these devices. This led him to use a formula simi-
 346 lar to the “R-factor,” which has been in use since before the mid-
 dle of last century (for example in [14]), and compares the
 experimental amplitudes with those derived from a model. His
 formula

$$R_{\text{sym}} = \frac{\sum_i \sum_{j=1}^{n_i} |I_j(hkl) - \bar{I}(hkl)|}{\sum_i \sum_{j=1}^{n_i} I_j(hkl)}$$

347 calculates the relative absolute deviation of intensity measurements
 348 from their mean value. R_{sym} has been in use since then, but renamed
 349 to R_{merge} probably because the formula can also be applied when
 350 merging symmetry-equivalent observations obtained from one or
 351 more crystals. R_{merge} measures the *precision* of the individual mea-
 352 surements (observations) of the intensities, and takes both the
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random and systematic error into account, in as far as the latter leads to differences in symmetry-related reflections.

It turned out that R_{merge} , as originally defined, has the flaw that for low multiplicity datasets it makes the precision appear to be better than it really is (by up to a factor of $\sqrt{2}$; [15]). This can be

fixed by including a factor $\sqrt{\frac{n_i}{n_i - 1}}$ in the numerator [15], and the

resulting precision indicator is called R_{meas} (or $R_{\text{r.i.m.}}$; [16]). Even though it more accurately reflects the precision of the data, it has unfortunately not been fully adopted by the crystallographic community; mainly, I assume, for the psychological reason that R_{meas} has a higher numerical value than R_{merge} .

Another measure of precision of intensities is the average I/σ ratio of the observations, $\langle I/\sigma \rangle_{\text{obs}}$. R_{meas} and $\langle I/\sigma \rangle_{\text{obs}}$ obey $R_{\text{meas}} \sim 0.8/\langle I/\sigma \rangle_{\text{obs}}$. This approximate relation is valid provided that the error model has been adjusted such that χ^2 , an indicator of the agreement between estimated and observed differences between symmetry-related observations, is near 1, and provided that $\langle I/\sigma \rangle$ is a good approximation of $\langle I/\sigma \rangle$. For a given dataset, these approximations are usually well fulfilled at high resolution. Nevertheless, the error models of different data processing programs usually yield quite different estimates of the $\sigma(hkl)$ and $\langle I/\sigma \rangle_{\text{obs}}$ values [17].

A precision indicator like R_{meas} or $\langle I/\sigma \rangle_{\text{obs}}$ is only useful in comparisons of datasets if the multiplicity of observations in the compared datasets is approximately the same. However, neither measure is useful for defining, for instance, a high-resolution cutoff since it does not take into account the obvious fact that multiple observations increase the precision.

The low-resolution R_{meas} value of a strongly exposed crystal mainly measures detectable systematic error and may therefore be considered another indicator of systematic error. This may explain the historical popularity of the related (but less suitable) R_{sym} value for (broadly) characterizing the “quality” of a dataset.

2.5 Indicators for the Precision of Merged Data

With one exception [18], merged data are used in all crystallographic calculations after the data processing step. Statistics referring to merged data are thus much more important than those referring to unmerged data, and ignorance or misunderstanding of this fact has led to common misconceptions about for example the choice data collection strategy, the choice of dataset to refine against, the possibility of merging of datasets, and about a suitable high-resolution cutoff [19].

In 1997, Diederichs and Karplus [15] therefore introduced a specially defined R -value which takes the multiplicity of observations into account, and calculates the precision of the merged data. This quantity, $R_{\text{mrgd-I}}$, measures the differences between merged

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intensities from two randomly selected subsets of the data. It is not in common use, but R_{split} ,

$$R_{\text{split}} = \frac{1}{\sqrt{2}} \frac{\sum_i |I_{\text{even}} - I_{\text{odd}}|}{\frac{1}{2} \sum_i I_{\text{even}} + I_{\text{odd}}}$$

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which is the same as $R_{\text{mrgd-1}}$ except for a factor of $1/\sqrt{2}$ and a simplified assignment of measurements to subsets, is in use by the Free Electron Laser community [20]. An equivalent quantity, $R_{\text{p.i.m.}}$ (“precision indicating merging R-factor”; [16])

$$R_{\text{pim}} = \frac{\sum_i \frac{1}{\sqrt{n_i - 1}} \sum_{j=1}^{n_i} |I_j(hkl) - \bar{I}(hkl)|}{\sum_i \sum_{j=1}^{n_i} I_j(hkl)}$$

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takes the \sqrt{n} improvement of precision directly into account, and has been shown to be useful when compared to R_{anom} which measures the precision of the anomalous signal.

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High-resolution R -values go to infinity when the signal vanishes [19]. This is obvious from the fact that the mean intensity, in the denominator of the formula, approaches zero in this situation, whereas the numerator approaches a constant which is determined by the variance of the background. This prevents the aforementioned data R -values from being useful for comparisons with model R -values at vanishing signal, where the latter approach a constant value [21]. As a consequence, data R -values are not suitable for defining a high-resolution cutoff, a little-known fact that has led to wrong conclusions for numerous datasets.

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One of the oldest and more useful estimators of precision is $\langle I/\sigma \rangle_{\text{mrgd}}$ (the subscript “mrgd” is added here only to distinguish it from $\langle I/\sigma \rangle_{\text{obs}}$; the subscript is not in common use) of the averaged data, which most data processing programs print out. There exists a reciprocal relationship between $\langle I/\sigma \rangle_{\text{mrgd}}$ and $R_{\text{mrgd-1}}/R_{\text{split}}/R_{\text{p.i.m.}}$ similar to the relation between $\langle I/\sigma \rangle_{\text{obs}}$ and R_{meas} . Unfortunately, the value of $\langle I/\sigma \rangle_{\text{mrgd}}$ depends on the error model, which usually varies significantly between different data processing programs [17]. Furthermore, for a given error model, $\langle I/\sigma \rangle_{\text{mrgd}}$ rises monotonously with higher multiplicity, even if the additional data are bad, e.g., in case of radiation damage. Nevertheless, historically a value of $\langle I/\sigma \rangle_{\text{mrgd}} > 2$ has been and continues to be used by many crystallographers as indicating the highest resolution shell that should be used for refinement [3, 22].

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The latest, statistically justifiable and so far most useful addition to the crystallographic data precision indicators is $CC_{1/2}$, which is derived from mainstream statistics and measures the

correlation coefficient between merged intensities obtained from two random subsets of the data. Its properties have been investigated recently [19, 23, 24]. It does not depend on estimated standard deviations of intensities, and its value is not misleadingly increased by important types of systematic errors [23]. Being a correlation coefficient, its value can be assessed for significance by a t -test, and most importantly, it offers the possibility to define a high-resolution cutoff based on the question “where do the data still have significant signal.”

Furthermore, from $CC_{1/2}$ we can calculate CC^* , a quantity on the same scale as correlation coefficients between measured intensities and F_{calc}^2 that are obtained from a model [19]. The latter, termed $CC_{\text{work}}/CC_{\text{free}}$, are defined for the “working” and the “free” set of reflections used in refinement, and should converge towards CC^* in the course of model completion and correction.

2.6 Accuracy of the Merged Data

Since the true intensity values are usually unknown and not measurable, in a strict sense it is impossible to estimate the accuracy of the merged intensity values, because undetected systematic error may be present which has to be added to the error estimate corresponding to the precision of the merged data.

As discussed, a few sources of systematic error remain potentially undetected; most notable are absorption, diffuse scattering and detector nonlinearity. Experience suggests that the undetected systematic error in the merged data may be on the order of a few percent for a good experiment; this is the relative difference between observed data and calculated intensities seen in small-molecule experiments where a complete and accurate model of the structure is available. A more quantitative, but still conservative upper limit is the reciprocal of ISa : this estimate asserts that the undetected systematic error is unlikely to be higher than the detected systematic error.

It is important to realize that when adding independent errors or error estimates, error propagation tells us that we have to add their squares, and finally take the square root. To give an example: suppose we expect an undetected relative systematic error of 3 % (conservative upper limit at $ISa = 33$), and a detected relative systematic error of 1.5 % (corresponding to $ISa = 33$ and fourfold multiplicity). In a low-resolution shell of a crystal, the random error in the merged data may amount to 2 %. We then have a relative accuracy estimate of about 4 % ($\sqrt{(0.03^2 + 0.015^2 + 0.02^2)} = 0.039$). In a resolution shell with 20 % relative random error in the merged data, we have a relative accuracy estimate of slightly more than 20 % ($\sqrt{(0.03^2 + 0.015^2 + 0.20^2)} = 0.203$). Thus, in practice, the estimate of the undetected error dominates the accuracy estimate of the strong low-resolution data, whereas for weak high-resolution data, the accuracy estimate is determined mostly by the precision of the merged data.

484 **2.7 Completeness**

485 All reflections in a dataset contribute to any place in the Fourier
486 synthesis of the electron density. In principle, this means that the
487 quality of the electron density map is compromised if not all reflec-
488 tions are measured. Quantitatively, since the reflections contribute
489 to the map in proportion to their amplitude, it is clear that the
490 strongest reflections are most important. Strong reflections are
491 found mainly at low resolution, where completeness fortunately is
492 favoured by the geometry of the diffraction experiment, i.e., the
493 low-resolution shells are usually more complete than the high-
494 resolution shells. Then again, the strongest reflections are those
495 that are most easily lost due to detector overload, which means
496 that another dataset (“low resolution pass”) may be needed to fill
497 in the missing (that is, saturated and thus inaccurate) reflections.

498 There exist no hard rules or studies about how incomplete
499 data may be to be still useful. If non-crystallographic symmetry is
500 present, averaging of the electron density maps of the copies of the
501 molecule may be performed, which partly substitutes for missing
502 completeness by virtue of redundancy in the unique dataset. If only
503 a single copy of the molecule resides in the asymmetric unit, a low-
504 resolution completeness of less than 75 % can be expected to lead
505 to quite noticeable degradation (artifacts) of maps (for an example,
506 see [http://ucxray.berkeley.edu/~jamesh/movies/completeness.
507 mpeg](http://ucxray.berkeley.edu/~jamesh/movies/completeness.mpeg)); also, data missing systematically in a region of reciprocal
508 space leads to more noticeable defects in the electron density than
509 randomly missing data ([http://www.ysbl.york.ac.uk/~cowtan/
510 fourier/duck4.html](http://www.ysbl.york.ac.uk/~cowtan/fourier/duck4.html)). On the other hand, if the low resolution
511 is almost complete, there is no reason to discard high-resolution
512 shells just for lack of completeness. To the contrary: all measured
513 reflections are valuable as they mitigate Fourier ripples, contribute
514 to the fine details in the electron density map, and constitute use-
515 ful restraints in refinement. The common practice of discarding
516 high-resolution data if their completeness is not “high enough” is
questionable, and has never been carefully tested.

517 **3 How to Obtain the Best Data from XDS**

518 The procedures for processing data with XDS have been described
519 [4, 5] and are not repeated here. Instead, based on first hand expe-
520 riences when processing datasets from my own group and helping
521 others with their challenging datasets, I focus on those steps that
522 are critical for data quality. For simplicity, we assume that a given
523 dataset can be indexed in the correct space group.

524 The overarching rules for data processing, in the order of their
525 importance, are that

- 526 (a) Sources of systematic error should be excluded if possible.
- 527 (b) The impact of any remaining sources of systematic error on
528 the data should be minimized.

- (c) The random error should be minimized. 529
- (d) The completeness of the data should be maximized. 530

Experience shows that goals (a), (b), and (c) are not conflict- 531
ing, but can be met with the same set of processing parameters. 532
Topic (d), however, requires a compromise. For instance, rejecting 533
the final frames of a dataset, in order to minimize the impact of 534
radiation damage, will reduce the completeness, or at least the 535
multiplicity of the data. Likewise, too generous masking of shad- 536
owed detector regions might lead to rejection of well-measured 537
reflections. 538

Analysis of the information provided by XDS (see below) may 539
lead to deeper insight about the data collection experiment itself. 540
Designing and evaluating an experiment is a genuinely scientific 541
approach and can and should not be left to automatic procedures. 542

The goal of data processing is to best parameterize the data 543
collection experiment. If the data processing is repeated with 544
changed parameters, the magnitude of the systematic error should 545
be monitored, using ISa. By *optimizing* (maximizing) ISa, indica- 546
tors of data precision are usually enhanced along the way, mainly 547
because the location and shape of the reflections on the frames 548
can be predicted more accurately. Generally, if the systematic 549
error in the data is reduced, the noise associated with it is con- 550
verted to signal. In case of doubt about any specific aspect of data 551
processing, the parameter value that maximizes ISa is usually the 552
correct one. 553

3.1 General Approach

To discover problems associated with data processing, it is essential 554
that in particular the files FRAME.cbf, INTEGRATE.LP, XDS_ 555
ASCII.HKL and CORRECT.LP are analyzed. 556

FRAME.cbf should be inspected (Fig. 1) to find out whether 557
spot shapes are regular, or whether there is indication of splitting 558
and multiple lattices. Irregular and split spots indicate problems in 559
crystal growth or handling, and always compromise data quality 560
due to higher random noise (because spots extend over more pix- 561
els) and higher systematic error (because the reflection profiles dif- 562
fer from the average). Furthermore, FRAME.cbf allows finding 563
out if predicted and observed diffraction patterns match. If they do 564
not, the space group or geometric parameters may be wrong which 565
may either prevent data processing from giving useful data, or may 566
lead to downstream problems in phasing and refinement. However, 567
this is beyond the scope of this article. Finally, FRAME.cbf, which 568
visualizes the last frame processed by INTEGRATE, should be 569
checked for the presence of ice rings (see below). 570

The tables in INTEGRATE.LP should be inspected for jumps 571
or large changes in frame-wise parameters like scale factors, mosa- 572
icity, beam divergence, or refined parameters like unit cell param- 573
eters, direct beam position, and distance (Fig. 2). Such changes 574

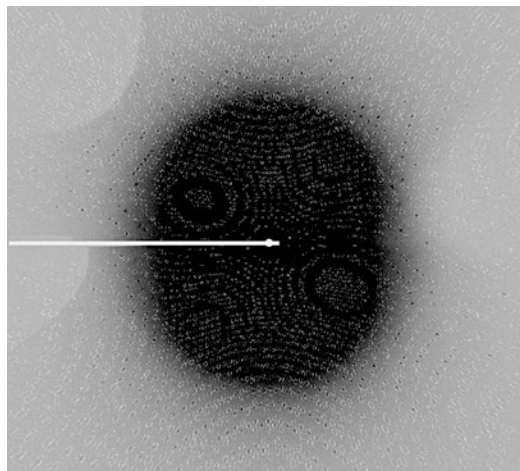


Fig. 1 Visualization of FRAME.cbf with xds-viewer. Predicted reflections are encircled. Two areas that are *shaded* by the cryo nozzle are visible (*left*, and *upper left*) that the user has not masked, which compromises data quality. The raw data used to prepare the figures are from a sulfur-SAD experiment with cubic insulin [25]; they may be obtained from http://www.helmholtz-berlin.de/forschung/funkma/soft-matter/forschung/bessy-mx/tutorial/experiment-1_en.html

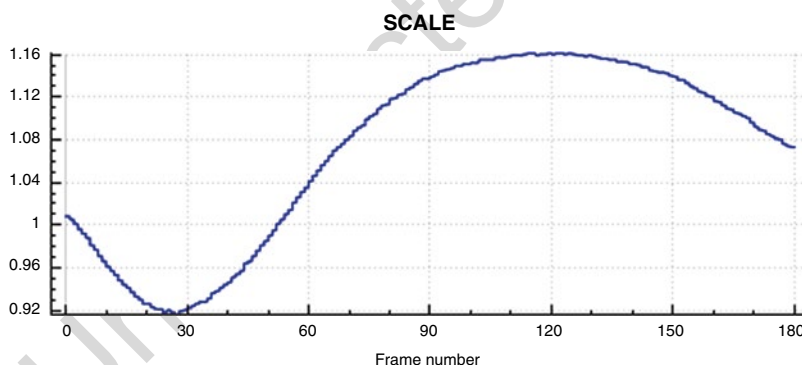
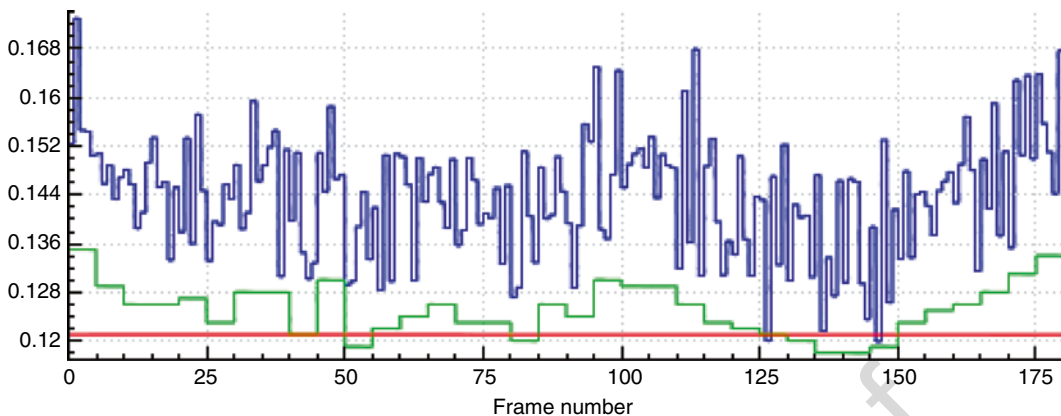
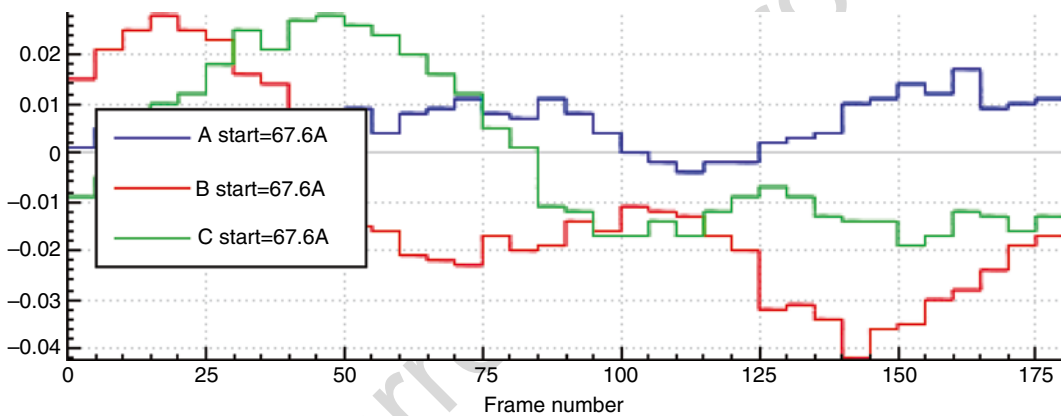


Fig. 2 Plots of some tabular quantities given in INTEGRATE.LP. Plot (a) shows scale factors, based on background pixels, of each frame. The plot is smooth which attests to the stability of the beam. Plot (b) shows different mosaicity estimates: for each frame (*blue*); for every 5° batch of data (*green*); for whole dataset (*red*). Due to the high symmetry, the curves are smooth. Plot (c) shows refined cell parameters, and (d) shows refined crystal setting angles. The variations in cell parameters do not follow a trend, which suggests that CELL should not be refined in INTEGRATE. The variations in setting angles are small; whether refinement of ORIENTATION in INTEGRATE improves the data should be tested, and the decision should be made based on Isa

SIGMAR



Change of Cell Axes [Å]



Crystal rotation [deg]

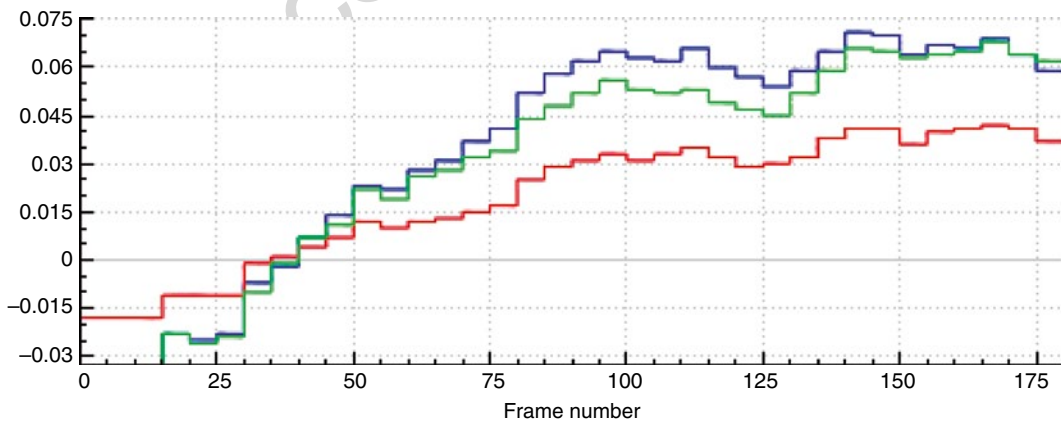


Fig. 2 (continued)

575 should be understood as indicating a potential source of systematic
576 error. Scale factor jumps should be brought to the attention of the
577 beamline manager; the other changes point to problems concern-
578 ing the experiment parameterization, like crystal decay or slippage,
579 and should trigger reprocessing after change of parameters like
580 DATA_RANGE, DELPHI, and REFINE(INTEGRATE) until no
581 further improvement can be obtained.

582 CORRECT.LP, among other statistics, reports on systematic
583 error (ISa) and the precision of unmerged and merged intensities
584 (R_{meas} and $CC_{1/2}$, respectively). It needs to be consulted to moni-
585 tor the success of changes to parameters in XDS.INP, and of
586 changes to the file XPARAM.XDS describing the geometry of the
587 experiment, which is used by INTEGRATE. It is useful to plot the
588 quantities reported in CORRECT.LP as a function of resolution,
589 and as a function of the upper frame range (Fig. 3).

590 XDSSTAT, a program that analyzes XDS_ASCII.HKL, should
591 be run and its output diverted to XDSSTAT.LP, to be visualized
592 with a plotting program. In addition, the control images written
593 by XDSSTAT offer a graphical way to inspect the projection of
594 several quantities on the detector surface, most notably R -values,
595 scale factors, and misfits (outliers identified during scaling) (Fig. 4).

596 Better processing may lead to a lower number of reflections
597 rejected during scaling. A guideline for the acceptable number of
598 outliers is the following: provided that the average multiplicity is 2
599 or higher, up to 1 % of the observations (the default that XDS
600 employs) may be rejected as outliers. If the percentage is higher,
601 the reason for this should be investigated, first by inspecting “mis-
602 fits.pck” as obtained from XDSSTAT. If “misfits.pck” shows con-
603 centric rings of outliers, the high percentage appears justified, but
604 the options for treating ice rings (see above) should be evaluated.
605 Second, if specific frames have many outliers, as shown by
606 XDSSTAT.LP, then these frames should possibly be omitted from
607 processing, and the reason why they delivered outlier data should
608 be investigated.

609 **3.2 Shaded Areas** 610 **of the Detector**

611 Several parameters have to be manually set before the inte-
612 gration step of XDS to mask shaded detector areas. Since
613 the keywords TRUSTED_REGION, UNTRUSTED_
614 RECTANGLE, UNTRUSTED_ELLIPSE, and UNTRUSTED_
615 QUADRILATERAL are not evaluated by the INTEGRATE and
616 CORRECT steps, they have to be specified earlier, namely, for the
617 INIT or DEFPIX steps. This requires graphical inspection of at
618 least a single data frame.

619 The low resolution limit of the data should be set such that the
620 shadow of the beam stop is completely excluded, using INCLUDE_
621 RESOLUTION_RANGE. Contrary to the keywords mentioned
622 before, this keyword can be specified at a later step (CORRECT).
If the lower resolution limit is too optimistic (i.e., too low), many

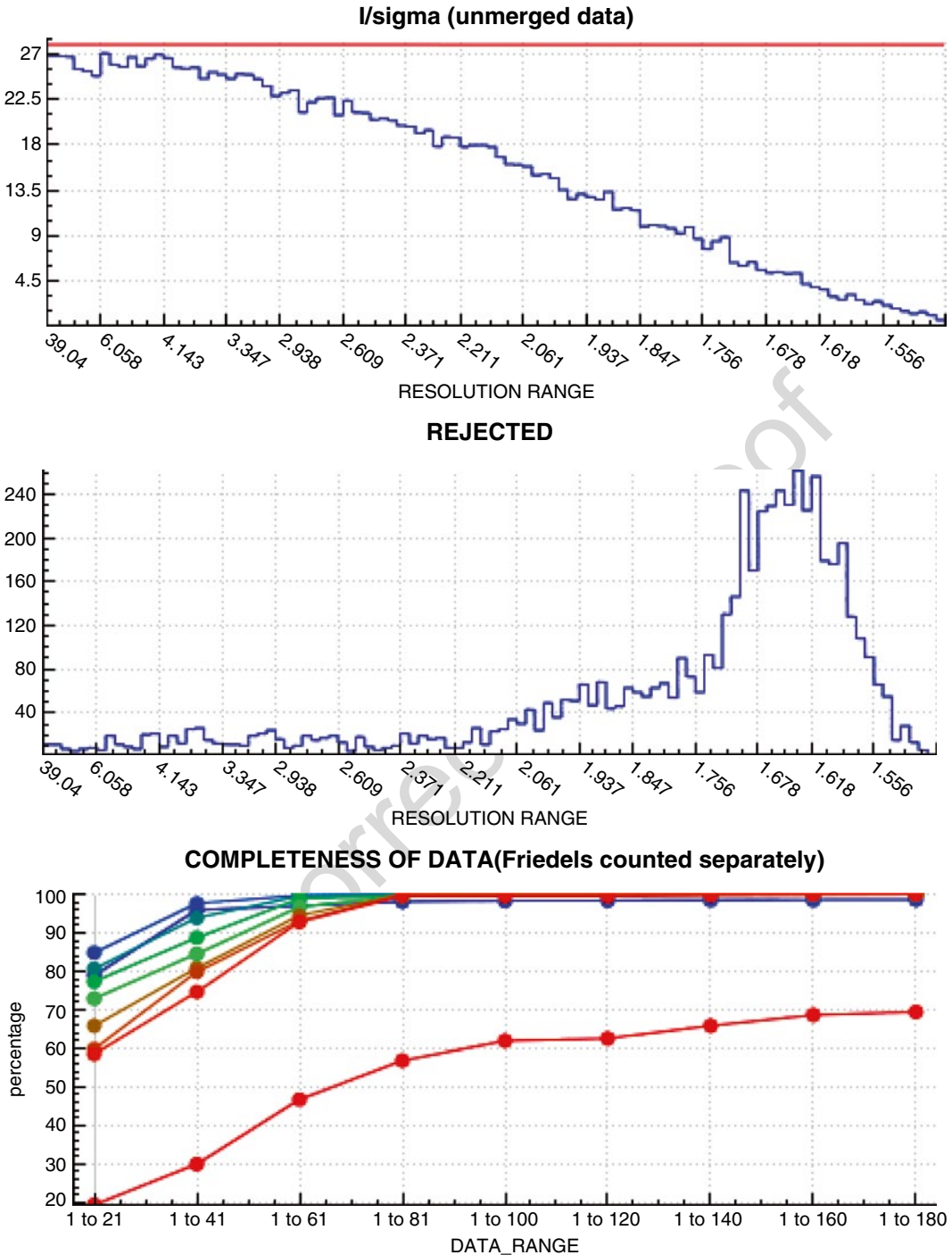
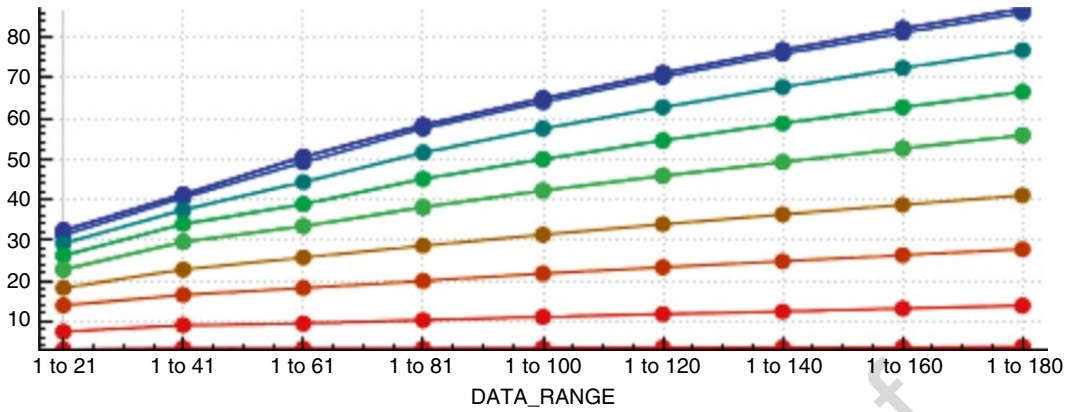
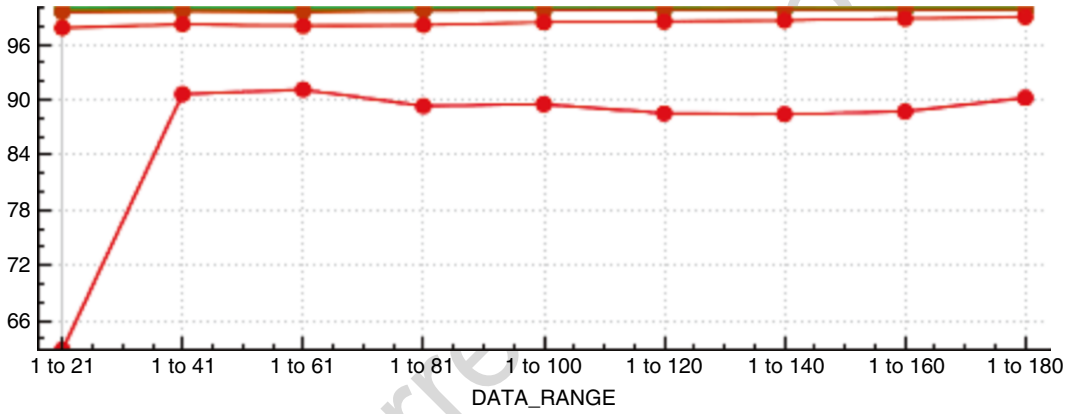


Fig. 3 Plots of some tabular quantities given in CORRECT.LP. Plot (a) shows $\langle I/\sigma_{\text{obs}} \rangle$ (blue) and I/σ_{a} (red), (b) the number of rejected observations. Both quantities are given as a function of resolution. Rejections peak at high resolution, due to the user's neglect of masking the shaded regions of the detector. The remaining plots show different quantities as a function of the number of frames, and thus of the multiplicity; the coloured curves (blue to red) correspond to different resolution ranges (low to high resolution): (c) completeness, (d) $\langle I/\sigma_{\text{mrgd}} \rangle$, (e) $CC_{1/2}$, and (f) CC_{anom} , the correlation coefficient between the anomalous signals obtained from half-datasets [26].

1/sigma (merged data)



CC(1/2)



Anomal Corr

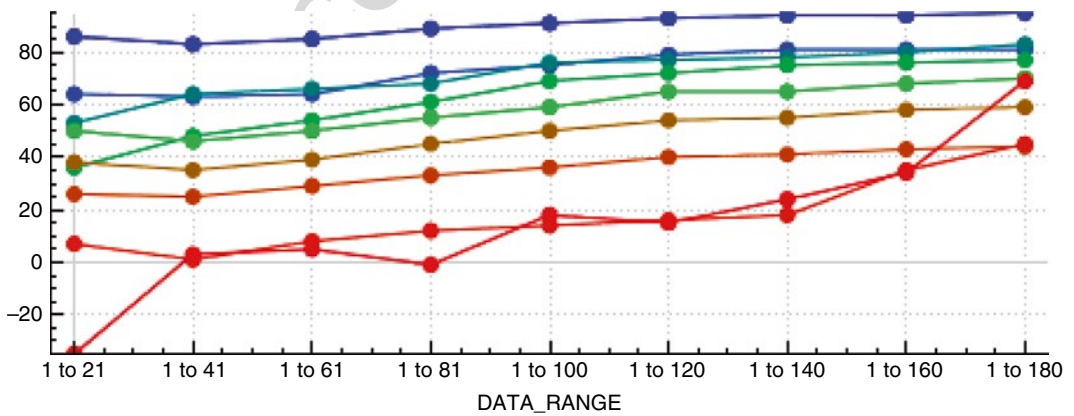


Fig. 3 (continued)

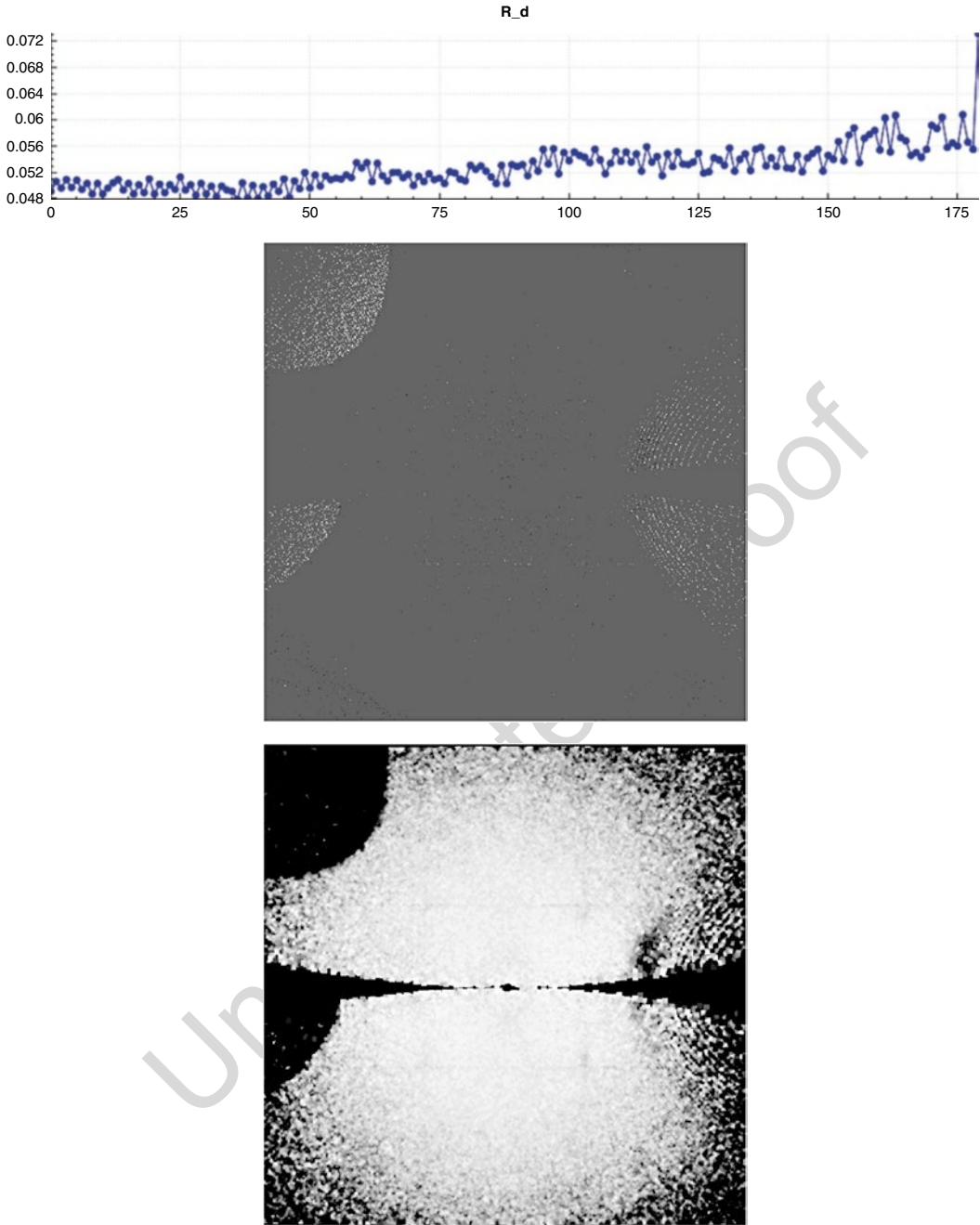


Fig. 4 Plots of some quantities obtained from XDSSTAT. Plot (a) shows a measure of radiation damage, R_d [27], revealing an increase in R_{meas} from 5.0 to 5.8 %. This corresponds to a 3 % contribution by radiation damage for the final frames of the dataset ($0.03^2 + 0.05^2 \sim 0.058^2$). Plot (b) shows the outliers projected on the detector; consistent with the high number of outliers revealed in Fig. 3b. Plot (c) displays R_{meas} projected on the detector, which reveals high values (dark areas) in the shaded areas

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rejections and high χ^2 values result in the low-resolution shell of the first statistics table available from CORRECT. If this is indeed observed, the lower resolution limit should be raised.

625 **3.3 Ice Rings,**
626 **Ice Reflections,**
627 **and "Aliens"**

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Single ice reflections, which fall onto a predicted spot position, are usually automatically excluded by the default outlier rejection mechanisms in CORRECT, either because their symmetry does not obey that of the macromolecular crystal, or because they are much stronger ("aliens" in CORRECT.LP) than the other reflections in their resolution range. The positions of rejected reflections can be visualized by inspecting the file "misfits.pck" using XDS-Viewer or adxv.

Strong ice rings should be manually excluded using EXCLUDE_RESOLUTION_RANGE; weak ice rings should be left to the automatic mechanisms for outlier rejection, because that results in higher completeness. To decide whether an ice ring should be considered strong or weak, the user should inspect the first statistics table in CORRECT.LP ("STANDARD ERROR OF REFLECTION INTENSITIES AS FUNCTION OF RESOLUTION"); ice rings are easily identified by a large number of rejections at resolution values near those of ice reflections (3.897, 3.669, 3.441, 2.671, 2.249, 2.072, 1.948, 1.918, 1.883, 1.721 Å for hexagonal ice, the form most often encountered). If the χ^2 and R -values in these resolution ranges are much higher than in the other ranges, the user should consider to reject the ice rings, using EXCLUDE_RESOLUTION_RANGE. This should also be done if the control image "scales.pck" (written by XDSSTAT) shows a significant deviation of scale factors from the value of 100 % at resolution values close to those of ice rings, or if "rf.pck" shows high R -values.

At very high resolution, in shells with mean intensity approaching zero, the "alien" identification algorithm sometimes rejects very many reflections when using its default value of REJECT_ALIEN=20. If this happens, the default should be raised to, say, 100 to prevent this from happening.

656 **3.4 Specific**
657 **Procedures**
658 **for Optimizing**
659 **Data Quality**

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Since the defaults in XDS.INP are carefully chosen and XDS has robust routines, very good data are usually obtained from a single processing run, in particular from good crystals. However, in case of difficult or very important datasets, the user may want to try and optimize the data processing parameters. This can be understood as minimizing or eliminating the impact of systematic errors introduced by the data processing step.

Three simple options should be tried:

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- (a) The globally optimized geometric parameter file GXPARM. XDS (obtained from CORRECT) may be used for another run of INTEGRATE and CORRECT. This operation may

reduce the systematic error which arises due to inaccurate 667
geometric parameters. It requires that the values of 668
“STANDARD DEVIATION OF SPOT POSITION” and 669
“STANDARD DEVIATION OF SPINDLE POSITION” in 670
CORRECT.LP are about as high as the corresponding values 671
printed out multiple times in INTEGRATE.LP, for each batch 672
of frames. This option is particularly successful if the SPOT_ 673
RANGE for COLSPOT was chosen significantly smaller than 674
the DATA_RANGE, because in that case the accuracy of geo- 675
metric parameters from IDXREF may not be optimal. 676

- (b) In XDS.INP, the averages of the refined profile-fitting param- 677
eters as printed out in INTEGRATE.LP, may be specified for 678
another run of INTEGRATE and CORRECT. Essentially, this 679
option attempts to minimize the error associated with poorly 680
determined spot profiles. This is most effective if there are few 681
strong reflections and/or large frame-to-frame variations 682
between estimates of SIGMAR (mosaicity) and SIGMAB 683
(beam divergence) as listed in INTEGRATE.LP. 684
- (c) In XDS.INP, one may specify the keyword REFINE 685
(INTEGRATE) with fewer (e.g., only ORIENTATION) or 686
no geometric parameters, instead of the default parameters 687
DISTANCE BEAM ORIENTATION CELL. This approach, 688
which also requires at least one more run of INTEGRATE and 689
CORRECT, is most efficient if the refined parameters, as 690
observed in previous INTEGRATE runs, vary randomly 691
around a mean value. Of course, preventing refinement of a 692
parameter is not the correct approach if its change is required 693
to achieve a better fit between observed and predicted reflec- 694
tion pattern. If removal of certain geometric parameters from 695
geometry refinement in INTEGRATE indeed improves ISa, 696
this indicates that the geometry refinement is not well enough 697
determined to improve them beyond those obtained by the 698
global refinement in IDXREF or CORRECT. This option 699
thus reduces the systematic error due to poorly determined 700
geometry. An alternative to switching refinement off is to 701
specify a larger DELPHI than the default (5°). 702

Ideally, each of the three options (a–c) should be tried sepa- 703
rately. Those options that improve ISa can then be tried in 704
combination, and the optimization procedure may be iterated as 705
long as there is significant improvement (of, say, a few percent) 706
in ISa. 707

In my experience, optimization may lead to significantly better 708
data, as shown by improved high-resolution $CC_{1/2}$ and improved 709
merging with other datasets, particularly for poor datasets with 710
high mosaicity and/or strong anisotropy. 711

712 **3.5 Don'ts**

Two possible ways of misusing XDS parameters should be mentioned.

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714 First, it may be tempting to increase the number of outliers
715 and thereby to “improve” (or rather “beautify”) the numerical val-
716 ues of quality indicators. This could in principle be achieved by
717 lowering the WFAC1 parameter below its default of 1. However,
718 the goal of data processing is to produce an accurate set of intensi-
719 ties for downstream calculations, not a set of statistical indicators
720 that have been artificially “massaged.” Experience shows that
721 reducing WFAC1 below its default almost always results in data
722 with worse accuracy; conversely, raising WFAC1 may sometimes
723 be a way to prevent too many observations to be rejected as outli-
724 ers. Only if there is additional evidence for the validity of reducing
725 WFAC1 should this quantity be lowered.

726 The second way to misuse XDS is to consider all the reflections
727 listed as “aliens” in CORRECT.LP as outliers, and to place them
728 into the file REMOVE.HKL to reject them in another CORRECT
729 run. This is not appropriate; it should only be done if there is addi-
730 tional evidence that these reflections are indeed outliers. Such evi-
731 dence could be the fact that the “aliens” occur at resolution values
732 corresponding to ice reflections (see above).

733 **3.6 High-
734 Resolution Cutoff**

The correct choice of high-resolution cutoff need not be made just
735 once, but can be made at various times during a crystallographic
736 study. The first is during data processing, and the additional times
737 are when the data are used for calculations such as molecular
738 replacement or model refinement, or calculating anomalous differ-
739 ence maps.

740 At the data processing stage, $CC_{1/2}$ should be used as the sole
741 indicator to determine a generous cutoff—one that avoids reject-
742 ing potentially useful data. It appears prudent not to discard reso-
743 lution shells with $CC_{1/2}$ larger than, say, 10 %, and it would appear
744 useful to deposit all of these data into the PDB, to enable later re-
745 refinement with refinement programs that can extract more infor-
746 mation from weak data.

747 For further crystallographic calculations, one must decide
748 upon the best cutoff to use for each application. Sometimes, as for
749 molecular replacement, all one desires is a successful solution and a
750 variety of choices may all work well. During the final model refine-
751 ment stage, when the goal is to get the most accurate model pos-
752 sible, a recent suggestion is that one need not make this decision
753 blindly, but that several high-resolution cutoffs can be compared
754 using the “pairwise refinement technique” [19, 23], to find the
755 high-resolution cutoff that delivers the best model under the given
756 circumstances: data, starting model, refinement strategy, and
refinement program.

4 The Relation of Data and Model Errors

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An atomic model of a macromolecule has to fulfil certain geometric restraints (bond lengths, angles, dihedrals, planes, van der Waals distances) because all macromolecules obey the same physico-chemical principles and consist of the same building blocks whose stereochemistry and physical properties are well known from high-resolution structures. Given a suitable starting model, these restraints leave several degrees of freedom that can be used, by a refinement program, to fit the experimental data.

Only recently has it been possible to connect data quality to model quality [19], which requires definition of suitable indicators, most notably CC^* , CC_{work} , and CC_{free} (see Subheading 2.5). The advantage of using correlation coefficients on intensities to measure both, the agreement of the observed intensities with the (unmeasurable!) true (ideal) intensities using CC^* , and the agreement of the observed intensities with the model intensities using CC_{work} and CC_{free} , lies in the fact that these correlation coefficients are comparable since they are defined in a consistent way—other than is the situation with $R_{\text{work}}/R_{\text{free}}$ and (e.g.) R_{meas} or $R_{\text{p.i.m.}}$.

Importantly, refinement should not result in CC_{work} being numerically higher than CC^* since that would mean that the model intensities agree better with the measured intensities than the ideal (true) intensities do. Since the measured intensities differ from the true intensities by noise, that would mean that the model fits the noise in the data, a situation that is called “overfitting.”

Since refinement makes CC_{work} approach CC^* , CC^* is a meaningful upper limit for CC_{work} . Any improvement in the data (from better processing or a new experiment) that results in higher CC^* allows obtaining a better model, with a higher CC_{work} (and of course better $R_{\text{work}}/R_{\text{free}}$).

In the following, we introduce a simple graphical representation for the relation between experimental data, true data, and the data corresponding to several models. If n is the number of reflections in a dataset, an n -dimensional space can represent all possible combinations of intensities. The set of true intensities T is represented by a point in this space, and so is the set of starting model intensities M and the set of measured unique intensities D . The three points T , M , and D can be conveniently represented as points in a two-dimensional subspace (plane) of this n -dimensional space, and other datasets may be represented as projections on this plane. It is this two-dimensional plane which is shown in Fig. 5.

A possible distance measure in this space may be established by considering 1-CC (“Pearson distance”), where CC is the correlation coefficient between intensities of a pair of datasets. Any relevant value of CC yields a Pearson distance less than 1; values of CC lower than 0, giving a Pearson distance greater than 1, are not meaningful because they correspond to unrelated data and models.

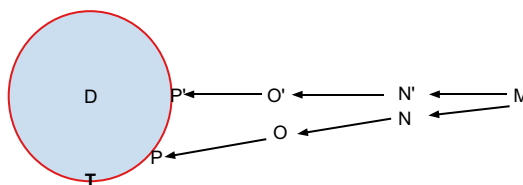


Fig. 5 Sketch of the relation between the intensities of the experimental data (D), the true (unmeasurable) data T, and those corresponding to various models. Arrows indicate the progression from a first starting model (M) to a final model P' (without restraints) or P (with restraints). Local minima of the refinement target give rise to the intermediate models N', O' and N, O, respectively

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In principle, there are infinitely many atomic models which fulfil the geometric restraints and could be used to calculate intensities. This means that the density of points in the plane that correspond to potential model structure factors is high everywhere. However, there is no smooth transition path, like that produced by refinement, between all these points. Nevertheless, they create local minima of the target function in refinement because these models fulfil the physicochemical restraints. In contrast, the subset of these local minima that are actually *biologically meaningful* is low overall, but high near T.

Since (by definition) CC^* is the correlation of the data with the true intensities, we realize that all points on the circle with radius $1-CC^*$ around D denote potential positions of the true intensities T. For the purposes of this discussion, one particular position of T at the lower edge of the circle has been marked. For the starting model M, a reasonable assumption is that the differences between M and D are not correlated with the error in D—after all, a model obtained by Molecular Replacement is oriented and translated based on the signal, not the error in D. Likewise, a map calculated from experimental phases is based on the signal in D; the error in D just produces noise in the map. As represented in the Figure, this means that the vector from D to M is approximately at right angle to the vector between D and T.

If no or weak restraints were applied, refinement of the starting model M would produce the sequence of models N', O', and P'—in other words, the intensities of the model would almost linearly approach those that were measured. However, applying the proper restraints adds information to the refinement which biases the model towards the truth; thus instead of N', O', and P' the model intensities are represented successively by (say) the local minima N, O, and P. The model, depending on the radius of convergence of the refinement protocol, needs to be manually adjusted to escape from these minima, and to progress towards T. Importantly, this only works if the starting model M is “close enough” to T; if it is not, manual adjustment becomes impossible

as the electron density maps are too poor, and at the same time there is too little biological meaning in the model to guide its manual improvement.

As soon as the circle around D is reached (near P) after manual corrections and restrained refinement, the desired change of model intensities further towards T is almost orthogonal to the direction towards D; thus, the model may easily become stuck in one of the many local minima on the arc, which fulfil the geometric restraints, are biologically meaningful, and represent similarly good CC_{work} values. This means that it becomes increasingly difficult to improve the model any further. After a few iterations without clear progress, crystallographers—subject to individual levels of experience and ambition—tend to abandon manual model correction and refinement. This explains why different crystallographers obtain different models from the same data. In any case, T is never reached, i.e., a residual error remains, but its amount depends on details of the refinement protocol and program, as well as on the amount of time and dedication that is invested into improvement of the model.

The following points are also noteworthy. First, if overfitting is avoided, the refined model P is outside or on the circle around D, because $1 - CC_{\text{work}} \geq 1 - CC^*$ due to $CC_{\text{work}} \leq CC^*$. If $CC_{\text{work}} = CC^*$, P lies on the arc between P' and T. One could argue that some overfitting could be tolerated as long as it reduces the distance between the refined model and T. Unfortunately, the latter distance cannot be measured, which is why it appears prudent to accept only little overfitting.

Second, the length of the arc between P' and T is proportional to $1 - CC^*$ which means that there are more local minima available for the refined model if the error in the data is higher. In reality, the space depicted as a one-dimensional arc in Figure N is a multidimensional one, and the number of local minima grows not only proportionally with $1 - CC^*$, but rather with a large exponent. Thus, a large family of similar models with indistinguishable quality may be obtained, simply by varying some refinement parameters, or displacing the coordinates a few tenths of an Angström during manual adjustment.

Third, larger random and systematic errors will lead to a larger radius of the circle. The average distance between T and those points on the circle that correspond to refined models depends linearly on the radius, which emphasizes that better data produce better models. Undetected systematic errors may lead to T being outside the circle, which means that refinement will not be able to push the model as close to T as when T is on the circle, demonstrating that it is important to detect and minimize systematic errors.

Fourth, the refined model P can be closer to T than to D which means that—somewhat counter intuitively at first!—the final model is actually *better* than the data. This is trivially true if the starting

887 model M happens to be close to T, but actually it is even the
 888 expected result, because judicious refinement and manual adjust-
 889 ment of a model takes sources of information beyond the mere
 890 experimental data restraints into account.

891 These considerations may illuminate the relation between data
 892 and model, and demonstrate that understanding and eliminating
 893 the sources of errors in the data helps in improving the atomic
 894 models on which our biological insight relies.

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