Chapter 10

Crystallographic Data and Model Quality

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Abstract

This article gives a consistent classification of sources of random and systematic errors in crystallographic data, and their influence on the averaged dataset obtained from a diffraction experiment. It discusses the relation between precision and accuracy and the crystallographic indicators used to estimate them, as well as topics like completeness and high-resolution cutoff. These concepts are applied in the context of presenting good practices for data processing with a widely used package, XDS. Recommendations are given for how to minimize the impact of several typical problems, like ice rings and shaded areas. Then, procedures for optimizing the processing parameters are explained. Finally, a simple graphical expression of some basic relations between data error and model error is suggested.

Key words X-ray crystallography, Accuracy, Precision, Random errors, Systematic errors, Merged data, Unmerged data, Indicators

1 Introduction

In the last decades, crystallography has been highly successful in delivering structural information about proteins, DNA, and RNA, the substrates of life on earth. The resolution of the method is good enough to discern the three dimensional structure of these macromolecules at the atomic level, which is essential to understand their diverse properties, functions and interactions. However, although it is easy to calculate the diffraction pattern for a given structure, the reverse task of deriving a molecular structure from just a single set of unique diffracted intensities is difficult, as the mathematical operation governing the former direction cannot be inverted in a unique way. To be solved experimentally, this “inverse problem,” or more specifically “phase problem,” requires more than just a single set of unique diffraction data. High quality of the data is a requirement for the experimental solution of the problem, but also for the refinement of the macromolecular structure, as discussed in Subheading 4.
The correct biological interpretation requires the best possible model of the macromolecule. To obtain the best model, every step of the structure determination procedure has to be performed in a close-to-optimal way. This means that the purification of the macromolecule, its crystallization, crystal handling, measurement of diffraction data, processing of the resulting datasets, and downstream steps such as structure solution, refinement, and validation each constitute scientific tasks that deserve specific attention, and have been undergoing continuous enhancements throughout the history of macromolecular crystallography.

Two kinds of numerical data are the result of a crystallographic experiment and usually deposited as such in the Protein Data Bank: the diffraction intensities as a reduced representation of the diffraction experiment, and the atomic coordinates resulting from the visual inspection and interpretation of electron density maps, and subsequent refinement. A third kind of numerical data, the raw data (frames) obtained in the diffraction experiment, have so far not been usually deposited in long-term archives, mainly due to (disk) space concerns. This is unfortunate since archiving of raw data would enable reprocessing of incorrectly processed data as well as enabling and taking advantage of future improvements in methodology, like extracting the diffuse scattering information.

The discussion focuses on data that correspond to a single atomic model. This rules out all the complications that arise from merging of non-isomorphous datasets, where each individual dataset corresponds to a different model—in this situation, a merged dataset would represent something like an average model, which violates the physicochemical requirements, and may not be biologically meaningful.

This chapter first presents the principles and concepts that need to be understood in the context of the rather broadly used term “data quality”; similar presentations may be found for example in refs. [1–3]. Second, the application of these principles to data processing with the XDS program package [4, 5], which the author is most familiar with, is explained. Third, data and atomic model are related in a graphical way, which allows some important and nontrivial conclusions to be drawn about how the former influence the latter.

2 Errors and Crystallographic Indicators

The goal of a crystallographic experiment is to obtain accurate intensities $I(hkl)$ for as many Bragg reflections $hkl$ as possible. I discuss two kinds of errors, random and systematic error, which exist in any experiment. A major difference between them is that the relative error arising from the random component decreases with increasing intensity, whereas the relative error in intensity
from the systematic component is (at least on average) \textit{constant},
often in the range between 1 and 10\%. A more specific description
for systematic error would thus be “fractional error,” but this
name is not in common use. Nonlinear errors also exist, but play
a minor role.

A well-designed crystallographic experiment has to strike an
appropriate compromise between the two kinds of error. For
example, a reduction of random error (see below) can be obtained
by longer or stronger exposure of the crystal, but this will inevi-
tably increase the systematic error from radiation damage to the
crystal. Ideally, the sum of both errors should be minimal, and
programs, e.g., “BEST” \cite{6}, exist that suggest a compromise, in
the form of a proposed “strategy” for the experiment. Fortunately,
the gradient of the sum of both errors is close to zero at and near
the optimal strategy, which means that small deviations from the
optimal strategy do not substantially decrease data quality.

The discussion of errors has to take the distinction between
precision and accuracy into account. The term “precision” refers
to the reproducibility of an experiment, and to the internal consis-
tency or relative deviation of the values obtained. For example, if
the number $e = 2.718\ldots$ should be determined in an experiment,
and two measurements would yield the values 3.217 and 3.219,
then these measurements are considered precise, because they
agree well with each other—their relative deviation is small.
However, they are not close to the true value—the error (or inac-
curacy) in their measurement amounts to about 0.5.

The term “accuracy,” on the other hand, refers to the devia-
tion of measured values from the true values. In this example, if
two measurements would yield the values 2.6 and 2.8, then the
results from this experiment are more accurate than that from
the previously mentioned experiment, although they are not as
precise.

Optimizing an experiment for precision alone therefore does
not ensure accuracy; rather, equating accuracy with precision also
requires the absence of any kind of error that has not been taken
into account in the precision estimate. To estimate accuracy, we
thus need to quantify both the precision of the data, and the unde-
tected error (which usually requires some knowledge about the
true value obtained by other means). If both can be quantified, we
can estimate the accuracy as the absolute or relative error of a
measurement.

\subsection{2.1 Random Error}
The crystallographic experiment measures the number of photons
contributing to each detector pixel. These photons arise from
Bragg reflections, but also from background scatter. The number
of photons in each pixel is subject to random fluctuations. These
are due to the quantum nature of photons; there exists a certain
probability of emission of a given photon by the crystal into a given
pixel in a unit of time, and each photon’s emission into that pixel is independent from that of other photons. As a result, photon counts are governed by Poisson (counting) statistics, which mathematically means that the variance of the photon number is equal to the photon number itself. Furthermore, a CCD detector may contribute a random component (“read-out noise”) to the total photon count (pixel detectors are almost noise-free), which is also due to quantum fluctuations in the detector hardware and may be considered as additional background.

Data processing software essentially adds the counts of the pixels belonging to each reflection, and subtracts an estimate of the background in each pixel, to give $I(hkl)$, the intensity of the Bragg reflection. The variance of $I(hkl)$ may be calculated, using the rules of error propagation, from the known variances of each contribution to $I(hkl)$; its square root will be called $\sigma_0(hkl)$ in the following. For strong reflections, where the background is negligible, this procedure gives a precision, expressed as relative random error in $I(hkl)$, of $\sigma_0(hkl)/I(hkl) \approx 1/\sqrt{I(hkl)}$.

The relative amount of the random error may be reduced by repeating the experiment, and averaging the results of the individual experiments. As the laws of error propagation show, the precision of the estimate of the averaged intensity is improved by a factor of $\sqrt{n}$ over that of an individual measurement, if $n$ is the number of repeated experiments with independent errors. Thus, the precision of the averaged (also called “merged”) data may be high even if the precision of each individual observation is low.

The square root function appears both in the relative error of a photon count arising from Poisson statistics, and in the improvement of precision from averaging of multiple measurements. It is important to realize that the mathematical reasons for the occurrence of the square root differ. Nevertheless, the fact that the square root occurs in both situations means that the relative error is in principle the same whether a reflection is measured ten times and averaged, or measured just once, but with ten times stronger exposure or ten times as long.

A high number (multiplicity, sometimes called redundancy) of observations of each unique reflection, together with low exposure of each observation, is therefore equivalent in terms of the precision of the merged data, to an experiment in which each unique reflection is just measured once, but exposed proportionally stronger. Thus, if only random error is considered, there would be no reason to perform experiments with high multiplicity.

### 2.2 Systematic Error

The term “systematic error” summarizes all types of error that are not purely random in nature, and these are due to macroscopic physical or technical properties of the experimental setup, the crystal, and the processing of its data. For instance, systematic errors may arise from imperfect spot shapes (split crystal), radiation
damage, absorption differences due to crystal shape and mounting, shutter synchronization problems, imperfect detector calibration and inhomogeneity of detector sensitivity, shadowed parts of the detector, nonlinear or overloaded detector, vibrations for example due to the cryo stream or fluctuations of the primary X-ray beam, imperfect or inaccurate assumptions about geometric parameters and computational models applied in the data processing step, and to other problems that may be significant for a given experiment.

Systematic error may appear to be random if its cause is unknown or cannot be fully described or modelled, but contrary to random counting error, the change of a reflection’s intensity is usually (at least on average) proportional to the intensity itself—thus the term “fractional error.” For example, a fluctuation in beam intensity changes all intensity values by the same percentage; absorption in the crystal or loop changes intensities in proportion to their original value; and using, during data processing, a mosaicity value that is too low, or a summation (integration) area that is too small, will chop off a certain fraction of the intensity.

Contrary to random error, the relative error of a single observation is not decreased by higher flux or longer exposure. However, many kinds of systematic errors in a crystallographic experiment at least partially cancel out if multiple measurements are averaged. This is the case if the experiment samples the possible values of the error term multiple times, in an even (or at least random) and unbiased way. Examples are beam instability, shutter problems, and most aspects of detector non-ideality, except those that result in nonlinear response (e.g., overload). Their influence on the final averaged data is decreased by averaging of n independent observations, and indeed the reduction of error then follows the same \( \sqrt{n} \) rule as applies to random error.

These kinds of systematic error may thus be considered as benign: their influence on the merged data may be mitigated by distributing the total experimental time and dose over many observations, or by collecting multiple datasets [7]. It is therefore systematic error, not random error, that mandates the collection of data over more than the absolute minimum of rotation range required for obtaining complete data. However, it is important to realize that after a full turn of the spindle, all those systematic errors that depend on the geometry of the experiment will be exactly repeated. It is thus highly advisable to change the crystal setting on the goniostat after at most 360°.

If all or most observations of a unique reflection are systematically affected in the same or a similar way, their systematic errors are not independent, and averaging may not necessarily decrease the systematic difference between true and estimated intensity (the accuracy). Known or well understood effects may often be modelled by analytical or empirical formulas. If a model for the specific error type is available and appropriate, the systematic difference is
accounted for, and any remaining difference between intensities may become a useful signal. In this way, a systematic effect may become a part of an extended description of the experiment, and does no longer contribute to the experimental error.

An example for this is absorption by the crystal and its environment (loop, mother liquor)—if it can be properly modelled, its influence is compensated. However, in low-symmetry space groups, all symmetry-related reflections may systematically be weakened or strengthened in the same way. Since only those systematic errors that lead to systematic differences can be corrected, no information about the proper absorption correction is available in this case. Therefore, at least one additional dataset should be measured in a different orientation of the crystal. The systematic absorption difference between the two resulting datasets may then be detected and corrected in software. It should be noted that even if absorption is not corrected in the data processing stage, it can be approximately compensated by an overall anisotropic overall displacement parameter in the refinement stage. This parameter then should not be interpreted as its name suggests, but rather as a compensation factor for an experimental property.

Importantly, for strong reflections (low resolution), systematic error is usually higher than random error; the converse is true for weak reflections (high resolution), where the signal-to-noise ratio is usually dominated by the random error term. However, radiation damage, the most devastating kind of systematic error, is an exception to this rule. Radiation damage, which changes (and ultimately destroys) the structure of the macromolecule during the measurement, induces a systematic error that is not mitigated by averaging of multiple observations, because it results in intensity measurements that do not scatter around a true value, but rather, with increasing dose, deviate further and further from the true value—the intensity at the beginning of the experiment.

The detrimental influence of radiation damage has to be avoided to a degree that depends on the kind of experiment, and its desired goal. In recent years, there has been some progress in describing the relation between dose and its footprint on the macromolecule [8, 9]. Furthermore, the influence of radiation damage may be partially compensated by zero-dose extrapolation, a computational technique [10]. However, it should be noted that the relative change of intensities by radiation damage is biggest at high resolution, where the signal may be so weak (i.e., the individual measurements so imprecise) that zero-dose extrapolation becomes inaccurate.

### 2.3 An Indicator for the Systematic Error

The error estimates $\sigma(hkl)$ obtained during integration of observations, being only based on counting statistics, are lower than the actual differences between intensities of symmetry-related observations, because the latter include the differences due to systematic...
errors. To account for the full difference, and thus to arrive at more useful error estimates $\sigma(hkl)$, the scaling and merging procedures operating on multiple reflections inflate the estimated error using an empirical formula $[3, 11]$ that employs separate scale factors for the random and the systematic error. This modification of error estimates works quite well in practice, and the coefficients of the formula (the “error model”) can be used to obtain an estimate of the systematic error of the dataset. The resulting estimate can be expressed as an “asymptotic signal-to-noise ratio,” abbreviated ISa $[11]$. ISa gives the numerical value of $I/\sigma$ for a hypothetical, infinitely strong reflection in the dataset, after adjustment of its $\sigma$ by the new error model. If no systematic errors would exist, ISa would be infinite, since $\sigma$ would be, from Poisson counting statistics, just the square root of $I$. However, real experiments are never ideal, which is why ISa is finite. ISa does not depend on the random error, making it insensitive to for example crystal size, mosaicity, exposure time, and flux, and is thus on an absolute scale.

The author has processed many datasets from different synchrotron beamlines and detectors. Empirically, it is found that datasets from CCD detectors rarely have ISa bigger than 30; datasets from pixel array detectors may have ISa values about twice as high compared to datasets from the same crystals collected on CCD detectors. This means that the use of a pixel array detector at a very stable beamline in an experiment with a good crystal may result in down to half the systematic error, compared to a CCD detector, and demonstrates the importance of detector technology. Conversely, if a split crystal, or a dataset with strong radiation damage, is measured on a perfect beamline, ISa may be as low as 10 (i.e., meaning that even the strongest reflections in the dataset will have signal-to-noise less than 10). Likewise, a good crystal may give a low ISa when the experimental setup or beam suffers from instability, or the cryo stream makes the crystal vibrate.

Data processing programs implement simplified or idealized assumptions about the experiment, and thus may themselves contribute some systematic error. To investigate the magnitude and properties of the systematic error from data processing, the author wrote the program SIM_MX $[12]$ that allows to simulate complete datasets with specified amounts of random and systematic errors. A simulated dataset with only random error should ideally produce an ISa of infinity. However, the data processing program that the author is most familiar with (XDS), in this situation gives ISa well above 100. This means that if a good crystal is used to obtain a dataset, the overall systematic error, as measured by ISa, is mostly due to deficiencies of the experimental setup, and not due to shortcomings of the data processing program. Unfortunately, a general weakness of the analysis of systematic error is that the simple error model usually employed does not allow to identify and thus correctly model the exact source of systematic error.
Conceptually, it is almost always impossible to accurately measure the systematic error, since the true intensities are unknown and not measurable. Obviously, ISa underestimates the overall systematic error, as only those systematic errors that lead to systematic differences enter its calculation. In my research group, we usually find that upon scaling two or more datasets together, the new error model for each dataset, which is calculated by the scaling program (XSCALE), almost always leads to a reduction of its ISa. This is due to the fact that only by comparing datasets can some systematic errors be detected.

For isomorphous datasets (e.g., same crystal but different orientation) which are >90% complete, have average multiplicity of 3 and more, and have higher symmetry than triclinic, we find that the reduction of ISa is usually less than 10%. Therefore, the underestimation of the systematic error by ISa is usually minor.

If non-isomorphous crystals are scaled and merged and the error model is recalculated, the newly determined ISa values have to account for the systematic differences arising from differences in unit-cell parameters and crystal contents, and may then be much reduced.

2.4 Indicators for the Precision of Unmerged Data

In crystallography, indicators to describe aggregated properties of the data are necessary because the number of reflections is so large that it is prohibitive to inspect individual reflections. The availability of multiple observations (called “multiplicity” or “redundancy”) allows their precision to be measured.

Historically, $R_{\text{sym}}$ was proposed by Arndt [13] for analysis of data from the first electronic area detectors. At the time, his interest was to create an indicator for how reproducibly the data are measured with these devices. This lead him to use a formula similar to the “$R$-factor,” which has been in use since before the middle of last century (for example in [14]), and compares the experimental amplitudes with those derived from a model. His formula

$$R_{\text{sym}} = \frac{\sum_{i}^{n} \sum_{j=1}^{n} |I_{ij}(hkl) - \overline{I}(hkl)|}{\sum_{i}^{n} \sum_{j=1}^{n} I_{ij}(hkl)}$$

calculates the relative absolute deviation of intensity measurements from their mean value. $R_{\text{sym}}$ has been in use since then, but renamed to $R_{\text{merge}}$ probably because the formula can also be applied when merging symmetry-equivalent observations obtained from one or more crystals. $R_{\text{merge}}$ measures the precision of the individual measurements (observations) of the intensities, and takes both the
random and systematic error into account, in as far as the latter leads to differences in symmetry-related reflections.

It turned out that $R_{\text{merge}}$, as originally defined, has the flaw that for low multiplicity datasets it makes the precision appear to be better than it really is (by up to a factor of $\sqrt{2}$; [15]). This can be fixed by including a factor $\sqrt{n_i - 1}$ in the numerator [15], and the resulting precision indicator is called $R_{\text{meas}}$ (or $R_{\text{r.i.m.}}$; [16]). Even though it more accurately reflects the precision of the data, it has unfortunately not been fully adopted by the crystallographic community; mainly, I assume, for the psychological reason that $R_{\text{meas}}$ has a higher numerical value than $R_{\text{merge}}$.

Another measure of precision of intensities is the average $I/\sigma$ ratio of the observations, $\langle I/\sigma_{\text{obs}} \rangle$. $R_{\text{meas}}$ and $\langle I/\sigma_{\text{obs}} \rangle$ obey $R_{\text{meas}} \sim 0.8/\langle I/\sigma_{\text{obs}} \rangle$. This approximate relation is valid provided that the error model has been adjusted such that $\chi^2$, an indicator of the agreement between estimated and observed differences between symmetry-related observations, is near 1, and provided that $\langle I/\sigma \rangle$ is a good approximation of $\langle I \rangle/\langle \sigma \rangle$. For a given dataset, these approximations are usually well fulfilled at high resolution. Nevertheless, the error models of different data processing programs usually yield quite different estimates of the $\sigma(hkl)$ and $\langle I/\sigma_{\text{obs}} \rangle$ values [17].

A precision indicator like $R_{\text{meas}}$ or $\langle I/\sigma_{\text{obs}} \rangle$ is only useful in comparisons of datasets if the multiplicity of observations in the compared datasets is approximately the same. However, neither measure is useful for defining, for instance, a high-resolution cutoff since it does not take into account the obvious fact that multiple observations increase the precision.

The low-resolution $R_{\text{meas}}$ value of a strongly exposed crystal mainly measures detectable systematic error and may therefore be considered another indicator of systematic error. This may explain the historical popularity of the related (but less suitable) $R_{\text{sym}}$ value for (broadly) characterizing the “quality” of a dataset.

2.5 Indicators for the Precision of Merged Data

With one exception [18], merged data are used in all crystallographic calculations after the data processing step. Statistics referring to merged data are thus much more important than those referring to unmerged data, and ignorance or misunderstanding of this fact has led to common misconceptions about for example the choice data collection strategy, the choice of dataset to refine against, the possibility of merging of datasets, and about a suitable high-resolution cutoff [19].

In 1997, Diederichs and Karplus [15] therefore introduced a specially defined $R$-value which takes the multiplicity of observations into account, and calculates the precision of the merged data. This quantity, $R_{\text{merg-d}}$, measures the differences between merged
intensities from two randomly selected subsets of the data. It is not in common use, but $R_{\text{split}}$,

$$R_{\text{split}} = \frac{1}{\sqrt{2}} \frac{\sum_{i} |I_{\text{even}} - I_{\text{odd}}|}{\frac{1}{2} \sum_{i} I_{\text{even}} + I_{\text{odd}}}$$

which is the same as $R_{\text{mrgd-I}}$ except for a factor of $1/\sqrt{2}$ and a simplified assignment of measurements to subsets, is in use by the Free Electron Laser community [20]. An equivalent quantity, $R_{\text{p.i.m.}}$ ("precision indicating merging R-factor"; [16])

$$R_{\text{p.i.m.}} = \frac{\sum_{j} \frac{1}{n_{j}} \sum_{i=1}^{n_{j}} |I_{j}(hkl) - \bar{T}(hkl)|}{\sum_{j} \sum_{i=1}^{n_{j}} I_{j}(hkl)}$$

takes the $\sqrt{n}$ improvement of precision directly into account, and has been shown to be useful when compared to $R_{\text{anom}}$ which measures the precision of the anomalous signal.

High-resolution $R$-values go to infinity when the signal vanishes [19]. This is obvious from the fact that the mean intensity, in the denominator of the formula, approaches zero in this situation, whereas the numerator approaches a constant which is determined by the variance of the background. This prevents the aforementioned data $R$-values from being useful for comparisons with model $R$-values at vanishing signal, where the latter approach a constant value [21]. As a consequence, data $R$-values are not suitable for defining a high-resolution cutoff, a little-known fact that has led to wrong conclusions for numerous datasets.

One of the oldest and more useful estimators of precision is $\langle I/\sigma \rangle_{\text{mrgd}}$ (the subscript "mrgd" is added here only to distinguish it from $\langle I/\sigma \rangle_{\text{obs}}$; the subscript is not in common use) of the averaged data, which most data processing programs print out. There exists a reciprocal relationship between $\langle I/\sigma \rangle_{\text{mrgd}}$ and $R_{\text{mrgd-I}}/R_{\text{split}}/R_{\text{p.i.m.}}$, similar to the relation between $\langle I/\sigma \rangle_{\text{obs}}$ and $R_{\text{anom}}$. Unfortunately, the value of $\langle I/\sigma \rangle_{\text{mrgd}}$ depends on the error model, which usually varies significantly between different data processing programs [17]. Furthermore, for a given error model, $\langle I/\sigma \rangle_{\text{mrgd}}$ rises monotonously with higher multiplicity, even if the additional data are bad, e.g., in case of radiation damage. Nevertheless, historically a value of $\langle I/\sigma \rangle_{\text{mrgd}} > 2$ has been and continues to be used by many crystallographers as indicating the highest resolution shell that should be used for refinement [3, 22].

The latest, statistically justifiable and so far most useful addition to the crystallographic data precision indicators is $CC_{1/2}$, which is derived from mainstream statistics and measures the
correlation coefficient between merged intensities obtained from two random subsets of the data. Its properties have been investigated recently [19, 23, 24]. It does not depend on estimated standard deviations of intensities, and its value is not misleadingly increased by important types of systematic errors [23]. Being a correlation coefficient, its value can be assessed for significance by a $t$-test, and most importantly, it offers the possibility to define a high-resolution cutoff based on the question “where do the data still have significant signal.”

Furthermore, from $CC_{1/2}$ we can calculate $CC^*$, a quantity on the same scale as correlation coefficients between measured intensities and $F_{\text{calc}}^2$ that are obtained from a model [19]. The latter, termed $CC_{\text{work}}/CC_{\text{free}}$, are defined for the “working” and the “free” set of reflections used in refinement, and should converge towards $CC^*$ in the course of model completion and correction.

### 2.6 Accuracy of the Merged Data

Since the true intensity values are usually unknown and not measurable, in a strict sense it is impossible to estimate the accuracy of the merged intensity values, because undetected systematic error may be present which has to be added to the error estimate corresponding to the precision of the merged data.

As discussed, a few sources of systematic error remain potentially undetected; most notable are absorption, diffuse scattering and detector nonlinearity. Experience suggests that the undetected systematic error in the merged data may be on the order of a few percent for a good experiment; this is the relative difference between observed data and calculated intensities seen in small-molecule experiments where a complete and accurate model of the structure is available. A more quantitative, but still conservative upper limit is the reciprocal of $\text{ISa}$: this estimate asserts that the undetected systematic error is unlikely to be higher than the detected systematic error.

It is important to realize that when adding independent errors or error estimates, error propagation tells us that we have to add their squares, and finally take the square root. To give an example: suppose we expect an undetected relative systematic error of $3\%$ (conservative upper limit at $\text{ISa}=33$), and a detected relative systematic error of $1.5\%$ (corresponding to $\text{ISa}=33$ and fourfold multiplicity). In a low-resolution shell of a crystal, the random error in the merged data may amount to $2\%$. We then have a relative accuracy estimate of about $4\%$ ($\sqrt{(0.03^2 + 0.015^2 + 0.02^2)}=0.039$). In a resolution shell with $20\%$ relative random error in the merged data, we have a relative accuracy estimate of slightly more than $20\%$ ($\sqrt{(0.03^2 + 0.015^2 + 0.20^2)}=0.203$). Thus, in practice, the estimate of the undetected error dominates the accuracy estimate of the strong low-resolution data, whereas for weak high-resolution data, the accuracy estimate is determined mostly by the precision of the merged data.
2.7 Completeness

All reflections in a dataset contribute to any place in the Fourier synthesis of the electron density. In principle, this means that the quality of the electron density map is compromised if not all reflections are measured. Quantitatively, since the reflections contribute to the map in proportion to their amplitude, it is clear that the strongest reflections are most important. Strong reflections are found mainly at low resolution, where completeness fortunately is favoured by the geometry of the diffraction experiment, i.e., the low-resolution shells are usually more complete than the high-resolution shells. Then again, the strongest reflections are those that are most easily lost due to detector overload, which means that another dataset (“low resolution pass”) may be needed to fill in the missing (that is, saturated and thus inaccurate) reflections.

There exist no hard rules or studies about how incomplete data may be to be still useful. If non-crystallographic symmetry is present, averaging of the electron density maps of the copies of the molecule may be performed, which partly substitutes for missing completeness by virtue of redundancy in the unique dataset. If only a single copy of the molecule resides in the asymmetric unit, a low-resolution completeness of less than 75% can be expected to lead to quite noticeable degradation (artifacts) of maps (for an example, see http://ucxray.berkeley.edu/~jamesh/movies/completeness.mpeg); also, data missing systematically in a region of reciprocal space leads to more noticeable defects in the electron density than randomly missing data (http://www.ysbl.york.ac.uk/~cowtan/fourier/duck4.html). On the other hand, if the low resolution is almost complete, there is no reason to discard high-resolution shells just for lack of completeness. To the contrary: all measured reflections are valuable as they mitigate Fourier ripples, contribute to the fine details in the electron density map, and constitute useful restraints in refinement. The common practice of discarding high-resolution data if their completeness is not “high enough” is questionable, and has never been carefully tested.

3 How to Obtain the Best Data from XDS

The procedures for processing data with XDS have been described [4, 5] and are not repeated here. Instead, based on first hand experiences when processing datasets from my own group and helping others with their challenging datasets, I focus on those steps that are critical for data quality. For simplicity, we assume that a given dataset can be indexed in the correct space group.

The overarching rules for data processing, in the order of their importance, are that

(a) Sources of systematic error should be excluded if possible.

(b) The impact of any remaining sources of systematic error on the data should be minimized.
(c) The random error should be minimized.

(d) The completeness of the data should be maximized.

Experience shows that goals (a), (b), and (c) are not conflicting, but can be met with the same set of processing parameters. Topic (d), however, requires a compromise. For instance, rejecting the final frames of a dataset, in order to minimize the impact of radiation damage, will reduce the completeness, or at least the multiplicity of the data. Likewise, too generous masking of shadowed detector regions might lead to rejection of well-measured reflections.

Analysis of the information provided by XDS (see below) may lead to deeper insight about the data collection experiment itself. Designing and evaluating an experiment is a genuinely scientific approach and can and should not be left to automatic procedures.

The goal of data processing is to best parameterize the data collection experiment. If the data processing is repeated with changed parameters, the magnitude of the systematic error should be monitored, using ISa. By optimizing (maximizing) ISa, indicators of data precision are usually enhanced along the way, mainly because the location and shape of the reflections on the frames can be predicted more accurately. Generally, if the systematic error in the data is reduced, the noise associated with it is converted to signal. In case of doubt about any specific aspect of data processing, the parameter value that maximizes ISa is usually the correct one.

To discover problems associated with data processing, it is essential that in particular the files FRAME.cbf, INTEGRATE.LP, XDS_ASCII.HKL and CORRECT.LP are analyzed.

FRAME.cbf should be inspected (Fig. 1) to find out whether spot shapes are regular, or whether there is indication of splitting and multiple lattices. Irregular and split spots indicate problems in crystal growth or handling, and always compromise data quality due to higher random noise (because spots extend over more pixels) and higher systematic error (because the reflection profiles differ from the average). Furthermore, FRAME.cbf allows finding out if predicted and observed diffraction patterns match. If they do not, the space group or geometric parameters may be wrong which may either prevent data processing from giving useful data, or may lead to downstream problems in phasing and refinement. However, this is beyond the scope of this article. Finally, FRAME.cbf, which visualizes the last frame processed by INTEGRATE, should be checked for the presence of ice rings (see below).

The tables in INTEGRATE.LP should be inspected for jumps or large changes in frame-wise parameters like scale factors, mosaicity, beam divergence, or refined parameters like unit cell parameters, direct beam position, and distance (Fig. 2). Such changes
Fig. 1 Visualization of FRAME.cbf with xds-viewer. Predicted reflections are encircled. Two areas that are shaded by the cryo nozzle are visible (left, and upper left) that the user has not masked, which compromises data quality. The raw data used to prepare the figures are from a sulfur-SAD experiment with cubic insulin [25]; they may be obtained from http://www.helmholtz-berlin.de/forschung/funkma/soft-matter/forschung/bessy-mx/tutorial/experiment-1_en.html

Fig. 2 Plots of some tabular quantities given in INTEGRATE.LP. Plot (a) shows scale factors, based on background pixels, of each frame. The plot is smooth which attests to the stability of the beam. Plot (b) shows different mosaicity estimates: for each frame (blue); for every 5° batch of data (green); for whole dataset (red). Due to the high symmetry, the curves are smooth. Plot (c) shows refined cell parameters, and (d) shows refined crystal setting angles. The variations in cell parameters do not follow a trend, which suggests that CELL should not be refined in INTEGRATE. The variations in setting angles are small; whether refinement of ORIENTATION in INTEGRATE improves the data should be tested, and the decision should be made based on ISA.
Fig. 2 (continued)
should be understood as indicating a potential source of systematic error. Scale factor jumps should be brought to the attention of the beamline manager; the other changes point to problems concerning the experiment parameterization, like crystal decay or slippage, and should trigger reprocessing after change of parameters like DATA_RANGE, DELPHI, and REFINE(INTEGRATE) until no further improvement can be obtained.

CORRECT.LP, among other statistics, reports on systematic error (ISa) and the precision of unmerged and merged intensities ($R_{\text{meas}}$ and $CC_{1/2}$, respectively). It needs to be consulted to monitor the success of changes to parameters in XDS.INP, and of changes to the file XPARM.XDS describing the geometry of the experiment, which is used by INTEGRATE. It is useful to plot the quantities reported in CORRECT.LP as a function of resolution, and as a function of the upper frame range (Fig. 3).

XDSSTAT, a program that analyzes XDS_ASCII.HKL, should be run and its output diverted to XDSSTAT.LP, to be visualized with a plotting program. In addition, the control images written by XDSSTAT offer a graphical way to inspect the projection of several quantities on the detector surface, most notably $R$-values, scale factors, and misfits (outliers identified during scaling) (Fig. 4).

Better processing may lead to a lower number of reflections rejected during scaling. A guideline for the acceptable number of outliers is the following: provided that the average multiplicity is 2 or higher, up to 1% of the observations (the default that XDS employs) may be rejected as outliers. If the percentage is higher, the reason for this should be investigated, first by inspecting “misfits.pck” as obtained from XDSSTAT. If “misfits.pck” shows concentric rings of outliers, the high percentage appears justified, but the options for treating ice rings (see above) should be evaluated. Second, if specific frames have many outliers, as shown by XDSSTAT.LP, then these frames should possibly be omitted from processing, and the reason why they delivered outlier data should be investigated.

### 3.2 Shaded Areas of the Detector

Several parameters have to be manually set before the integration step of XDS to mask shaded detector areas. Since the keywords TRUSTED_REGION, UNTRUSTED_RECTANGLE, UNTRUSTED_ELLIPSE, and UNTRUSTED_QUADRILATERAL are not evaluated by the INTEGRATE and CORRECT steps, they have to be specified earlier, namely, for the INIT or DEFPIX steps. This requires graphical inspection of at least a single data frame.

The low resolution limit of the data should be set such that the shadow of the beam stop is completely excluded, using INCLUDE_RESOLUTION_RANGE. Contrary to the keywords mentioned before, this keyword can be specified at a later step (CORRECT). If the lower resolution limit is too optimistic (i.e., too low), many
Fig. 3 Plots of some tabular quantities given in CORRECT.LP. Plot (a) shows $I/\sigma_{\text{obs}}$ (blue) and ISa (red), (b) the number of rejected observations. Both quantities are given as a function of resolution. Rejections peak at high resolution, due to the user's neglect of masking the shaded regions of the detector. The remaining plots show different quantities as a function of the number of frames, and thus of the multiplicity; the coloured curves (blue to red) correspond to different resolution ranges (low to high resolution); (c) completeness, (d) $I/\sigma_{\text{mrgd}}$, (e) CC$_{1/2}$, and (f) CC$_{\text{anom}}$, the correlation coefficient between the anomalous signals obtained from half-datasets [26].
Fig. 3 (continued)
Fig. 4 Plots of some quantities obtained from XDSSTAT. Plot (a) shows a measure of radiation damage, $R_d$ [27], revealing an increase in $R_{\text{meas}}$ from 5.0 to 5.8 %. This corresponds to a 3 % contribution by radiation damage for the final frames of the dataset ($0.03^2 + 0.05^2 \sim 0.058^2$). Plot (b) shows the outliers projected on the detector; consistent with the high number of outliers revealed in Fig. 3b. Plot (c) displays $R_{\text{meas}}$ projected on the detector, which reveals high values (dark areas) in the shaded areas.
rejections and high $\chi^2$ values result in the low-resolution shell of the first statistics table available from CORRECT. If this is indeed observed, the lower resolution limit should be raised.

**3.3 Ice Rings, Ice Reflections, and “Aliens”**

Single ice reflections, which fall onto a predicted spot position, are usually automatically excluded by the default outlier rejection mechanisms in CORRECT, either because their symmetry does not obey that of the macromolecular crystal, or because they are much stronger ("aliens" in CORRECT.LP) than the other reflections in their resolution range. The positions of rejected reflections can be visualized by inspecting the file “misfits.pck” using XDS-Viewer or adxv.

Strong ice rings should be manually excluded using EXCLUDE_RESOLUTION_RANGE; weak ice rings should be left to the automatic mechanisms for outlier rejection, because that results in higher completeness. To decide whether an ice ring should be considered strong or weak, the user should inspect the first statistics table in CORRECT.LP ("STANDARD ERROR OF REFLECTION INTENSITIES AS FUNCTION OF RESOLUTION"); ice rings are easily identified by a large number of rejections at resolution values near those of ice reflections (3.897, 3.669, 3.441, 2.671, 2.249, 2.072, 1.948, 1.918, 1.883, 1.721 Å for hexagonal ice, the form most often encountered). If the $\chi^2$ and $R$-values in these resolution ranges are much higher than in the other ranges, the user should consider to reject the ice rings, using EXCLUDE_RESOLUTION_RANGE. This should also be done if the control image “scales.pck” (written by XDSSTAT) shows a significant deviation of scale factors from the value of 100% at resolution values close to those of ice rings, or if “rf.pck” shows high $R$-values.

At very high resolution, in shells with mean intensity approaching zero, the “alien” identification algorithm sometimes rejects very many reflections when using its default value of REJECT_ALIEN=20. If this happens, the default should be raised to, say, 100 to prevent this from happening.

**3.4 Specific Procedures for Optimizing Data Quality**

Since the defaults in XDS.INP are carefully chosen and XDS has robust routines, very good data are usually obtained from a single processing run, in particular from good crystals. However, in case of difficult or very important datasets, the user may want to try and optimize the data processing parameters. This can be understood as minimizing or eliminating the impact of systematic errors introduced by the data processing step.

Three simple options should be tried:

(a) The globally optimized geometric parameter file GXPARM. XDS (obtained from CORRECT) may be used for another run of INTEGRATE and CORRECT. This operation may
reduce the systematic error which arises due to inaccurate geometric parameters. It requires that the values of “STANDARD DEVIATION OF SPOT POSITION” and “STANDARD DEVIATION OF SPINDLE POSITION” in CORRECT.LP are about as high as the corresponding values printed out multiple times in INTEGRATE.LP, for each batch of frames. This option is particularly successful if the SPOT_RANGE for COLSPOT was chosen significantly smaller than the DATA_RANGE, because in that case the accuracy of geometric parameters from IDXREF may not be optimal.

(b) In XDS.INP, the averages of the refined profile-fitting parameters as printed out in INTEGRATE.LP, may be specified for another run of INTEGRATE and CORRECT. Essentially, this option attempts to minimize the error associated with poorly determined spot profiles. This is most effective if there are few strong reflections and/or large frame-to-frame variations between estimates of SIGMAR (mosaicity) and SIGMAB (beam divergence) as listed in INTEGRATE.LP.

(c) In XDS.INP, one may specify the keyword REFINE (INTEGRATE) with fewer (e.g., only ORIENTATION) or no geometric parameters instead of the default parameters DISTANCE BEAM ORIENTATION CELL. This approach, which also requires at least one more run of INTEGRATE and CORRECT, is most efficient if the refined parameters, as observed in previous INTEGRATE runs, vary randomly around a mean value. Of course, preventing refinement of a parameter is not the correct approach if its change is required to achieve a better fit between observed and predicted reflection pattern. If removal of certain geometric parameters from geometry refinement in INTEGRATE indeed improves ISa, this indicates that the geometry refinement is not well enough determined to improve them beyond those obtained by the global refinement in IDXREF or CORRECT. This option thus reduces the systematic error due to poorly determined geometry. An alternative to switching refinement off is to specify a larger DELPHI than the default (5°).

Ideally, each of the three options (a–c) should be tried separately. Those options that improve ISa can then be tried in combination, and the optimization procedure may be iterated as long as there is significant improvement (of, say, a few percent) in ISa.

In my experience, optimization may lead to significantly better data, as shown by improved high-resolution CC1/2 and improved merging with other datasets, particularly for poor datasets with high mosaicity and/or strong anisotropy.
Two possible ways of misusing XDS parameters should be mentioned.

First, it may be tempting to increase the number of outliers and thereby to “improve” (or rather “beautify”) the numerical values of quality indicators. This could in principle be achieved by lowering the WFAC1 parameter below its default of 1. However, the goal of data processing is to produce an accurate set of intensities for downstream calculations, not a set of statistical indicators that have been artificially “massaged.” Experience shows that reducing WFAC1 below its default almost always results in data with worse accuracy; conversely, raising WFAC1 may sometimes be a way to prevent too many observations to be rejected as outliers. Only if there is additional evidence for the validity of reducing WFAC1 should this quantity be lowered.

The second way to misuse XDS is to consider all the reflections listed as “aliens” in CORRECT.LP as outliers, and to place them into the file REMOVE.HKL to reject them in another CORRECT run. This is not appropriate; it should only be done if there is additional evidence that these reflections are indeed outliers. Such evidence could be the fact that the “aliens” occur at resolution values corresponding to ice reflections (see above).

The correct choice of high-resolution cutoff need not be made just once, but can be made at various times during a crystallographic study. The first is during data processing, and the additional times are when the data are used for calculations such as molecular replacement or model refinement, or calculating anomalous difference maps.

At the data processing stage, \( CC_{1/2} \) should be used as the sole indicator to determine a generous cutoff—one that avoids rejecting potentially useful data. It appears prudent not to discard resolution shells with \( CC_{1/2} \) larger than, say, 10 %, and it would appear useful to deposit all of these data into the PDB, to enable later refinement with refinement programs that can extract more information from weak data.

For further crystallographic calculations, one must decide upon the best cutoff to use for each application. Sometimes, as for molecular replacement, all one desires is a successful solution and a variety of choices may all work well. During the final model refinement stage, when the goal is to get the most accurate model possible, a recent suggestion is that one need not make this decision blindly, but that several high-resolution cutoffs can be compared using the “pairwise refinement technique” \([19, 23]\), to find the high-resolution cutoff that delivers the best model under the given circumstances: data, starting model, refinement strategy, and refinement program.
The Relation of Data and Model Errors

An atomic model of a macromolecule has to fulfill certain geometric restraints (bond lengths, angles, dihedrals, planes, van der Waals distances) because all macromolecules obey the same physicochemical principles and consist of the same building blocks whose stereochemistry and physical properties are well known from high-resolution structures. Given a suitable starting model, these restraints leave several degrees of freedom that can be used, by a refinement program, to fit the experimental data.

Only recently has it been possible to connect data quality to model quality [19], which requires definition of suitable indicators, most notably $CC^*$, $CC_{\text{work}}$, and $CC_{\text{free}}$ (see Subheading 2.5). The advantage of using correlation coefficients on intensities to measure both, the agreement of the observed intensities with the (unmeasurable!) true (ideal) intensities using $CC^*$, and the agreement of the observed intensities with the model intensities using $CC_{\text{work}}$ and $CC_{\text{free}}$, lies in the fact that these correlation coefficients are comparable since they are defined in a consistent way—other than is the situation with $R_{\text{work}}/R_{\text{free}}$ (and (e.g.,) $R_{\text{meas}}$ or $R_{\text{p.i.m.}}$).

Importantly, refinement should not result in $CC_{\text{work}}$ being numerically higher than $CC^*$ since that would mean that the model intensities agree better with the measured intensities than the ideal (true) intensities do. Since the measured intensities differ from the true intensities by noise, that would mean that the model fits the noise in the data, a situation that is called “overfitting.”

Since refinement makes $CC_{\text{work}}$ approach $CC^*$, $CC^*$ is a meaningful upper limit for $CC_{\text{work}}$. Any improvement in the data (from better processing or a new experiment) that results in higher $CC^*$ allows obtaining a better model, with a higher $CC_{\text{work}}$ (and of course better $R_{\text{work}}/R_{\text{free}}$).

In the following, we introduce a simple graphical representation for the relation between experimental data, true data, and the data corresponding to several models. If n is the number of reflections in a dataset, an n-dimensional space can represent all possible combinations of intensities. The set of true intensities T is represented by a point in this space, and so is the set of starting model intensities M and the set of measured unique intensities D. The three points T, M, and D can be conveniently represented as points in a two-dimensional subspace (plane) of this n-dimensional space, and other datasets may be represented as projections on this plane. It is this two-dimensional plane which is shown in Fig. 5.

A possible distance measure in this space may be established by considering 1-CC (“Pearson distance”), where CC is the correlation coefficient between intensities of a pair of datasets. Any relevant value of CC yields a Pearson distance less than 1; values of CC lower than 0, giving a Pearson distance greater than 1, are not meaningful because they correspond to unrelated data and models.
In principle, there are infinitely many atomic models which fulfil the geometric restraints and could be used to calculate intensities. This means that the density of points in the plane that correspond to potential model structure factors is high everywhere. However, there is no smooth transition path, like that produced by refinement, between all these points. Nevertheless, they create local minima of the target function in refinement because these models fulfil the physicochemical restraints. In contrast, the subset of these local minima that are actually biologically meaningful is low overall, but high near $T$.

Since (by definition) $CC^*$ is the correlation of the data with the true intensities, we realize that all points on the circle with radius $1-CC^*$ around $D$ denote potential positions of the true intensities $T$. For the purposes of this discussion, one particular position of $T$ at the lower edge of the circle has been marked. For the starting model $M$, a reasonable assumption is that the differences between $M$ and $D$ are not correlated with the error in $D$—after all, a model obtained by Molecular Replacement is oriented and translated based on the signal, not the error in $D$. Likewise, a map calculated from experimental phases is based on the signal in $D$; the error in $D$ just produces noise in the map. As represented in the Figure, this means that the vector from $D$ to $M$ is approximately at right angle to the vector between $D$ and $T$.

If no or weak restraints were applied, refinement of the starting model $M$ would produce the sequence of models $N'$, $O'$, and $P'$—in other words, the intensities of the model would almost linearly approach those that were measured. However, applying the proper restraints adds information to the refinement which biases the model towards the truth; thus instead of $N'$, $O'$, and $P'$ the model intensities are represented successively by (say) the local minima $N$, $O$, and $P$. The model, depending on the radius of convergence of the refinement protocol, needs to be manually adjusted to escape from these minima, and to progress towards $T$. Importantly, this only works if the starting model $M$ is “close enough” to $T$; if it is not, manual adjustment becomes impossible.
as the electron density maps are too poor, and at the same time there is too little biological meaning in the model to guide its manual improvement.

As soon as the circle around D is reached (near P) after manual corrections and restrained refinement, the desired change of model intensities further towards T is almost orthogonal to the direction towards D; thus, the model may easily become stuck in one of the many local minima on the arc, which fulfil the geometric restraints, are biologically meaningful, and represent similarly good $CC_{\text{work}}$ values. This means that it becomes increasingly difficult to improve the model any further. After a few iterations without clear progress, crystallographers—subject to individual levels of experience and ambition—tend to abandon manual model correction and refinement. This explains why different crystallographers obtain different models from the same data. In any case, T is never reached, i.e., a residual error remains, but its amount depends on details of the refinement protocol and program, as well as on the amount of time and dedication that is invested into improvement of the model.

The following points are also noteworthy. First, if overfitting is avoided, the refined model P is outside or on the circle around D, because $1 - CC_{\text{work}} \geq 1 - CC^*$ due to $CC_{\text{work}} \leq CC^*$. If $CC_{\text{work}} = CC^*$, P lies on the arc between P′ and T. One could argue that some overfitting could be tolerated as long as it reduces the distance between the refined model and T. Unfortunately, the latter distance cannot be measured, which is why it appears prudent to accept only little overfitting.

Second, the length of the arc between P′ and T is proportional to $1 - CC^*$ which means that there are more local minima available for the refined model if the error in the data is higher. In reality, the space depicted as a one-dimensional arc in Figure N is a multidimensional one, and the number of local minima grows not only proportionally with $1 - CC^*$, but rather with a large exponent. Thus, a large family of similar models with indistinguishable quality may be obtained, simply by varying some refinement parameters, or displacing the coordinates a few tenths of an Angström during manual adjustment.

Third, larger random and systematic errors will lead to a larger radius of the circle. The average distance between T and those points on the circle that correspond to refined models depends linearly on the radius, which emphasizes that better data produce better models. Undetected systematic errors may lead to T being outside the circle, which means that refinement will not be able to push the model as close to T as when T is on the circle, demonstrating that it is important to detect and minimize systematic errors.

Fourth, the refined model P can be closer to T than to D which means that—somewhat counter intuitively at first!—the final model is actually better than the data. This is trivially true if the starting
model M happens to be close to T, but actually it is even the
expected result, because judicious refinement and manual adjust-
ment of a model takes sources of information beyond the mere
experimental data restraints into account.

These considerations may illuminate the relation between data
and model, and demonstrate that understanding and eliminating
the sources of errors in the data helps in improving the atomic
models on which our biological insight relies.

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